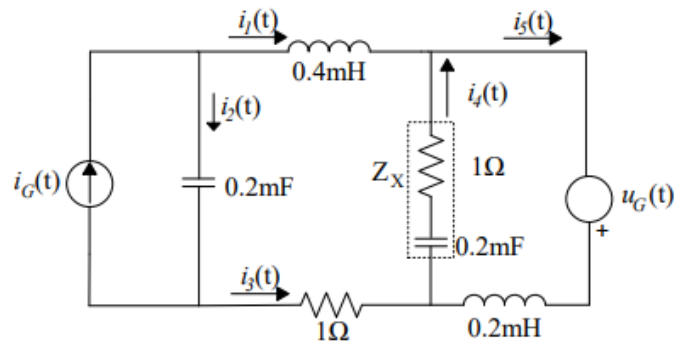


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3. U kolu sa slike, na kružnoj učestanosti  $\omega=10$  krad/s, uspostavljen je prostoperiodični režim i poznato je  $u_G(t) = 4V \cos(\omega t + 45^\circ)$  i  $i_G(t) = 1A \sin(\omega t)$ . Potrebno je izračunati:

- [4] impedansu  $Z_x$ ;
- [18] struje  $I_{1.5}$  u kompleksnom domenu za referentne smerove označene na slici;
- [18] struje  $i_{1.5}(t)$  u vremenskom domenu za referentne smerove označene na slici
- [10] aktivnu, reaktivnu i kompleksnu snagu koja se razvija na impedansi  $Z_x$ .



$u_G(t) = 4V \cos(\omega t + 45^\circ)$      $U_G = 2\sqrt{2} \angle 45^\circ$   
 $i_G(t) = 1A \sin(\omega t + 45^\circ)$      $I_G = \frac{\sqrt{2}}{2} \angle -90^\circ$   
 $\omega = 10 \cdot 10^3 \text{ rad/s}$

a)  $Z_x = 1\Omega \cdot j \frac{1}{10 \cdot 10^{-6}} = 1\Omega - j \frac{1}{2}\Omega = (1 - j \frac{1}{2})\Omega$

6)  $U_B = 2\Omega \cdot (1 + j \frac{1}{2}) = (2 + j2)V$ ,  $I_6 = \frac{\sqrt{2}}{2} \angle -j = -j \frac{\sqrt{2}}{2}$ ,  $X_{\text{cap}} = -j \frac{1}{2}\Omega$ ,  $X_{\text{ind}} = j4\Omega$ ,  $X_{\text{ind}} = j2\Omega$

$U_1 = \left( \frac{1}{1+j3.5} + \frac{1}{1-j0.5} + \frac{1}{j2} \right) = \frac{-\sqrt{2}}{4j3.5} - \frac{2+j2}{2j}$   
 $U_2 = \left( \frac{1-j3.5}{1+j3.5} + \frac{1+j0.5}{1-j0.5} - \frac{j}{2} \right) = \frac{-\sqrt{2}}{4+j4} + j(1+j)$   
 $U_3 = \left( \frac{1-j3.5}{1.25} + \frac{1+j0.5}{1.25} - \frac{j}{2} \right) = \frac{-2 \cdot (4-j4)}{16+j16} + j(1+j)$

$U_4 = \frac{4-j4}{5} + \frac{4+j2}{5} - 0.5j = \frac{-\sqrt{2} \cdot 4 - j4\sqrt{2}}{2.12} + j(1+j)$   
 $U_5 = \frac{16-j56}{2.12} + \frac{8\sqrt{2} + j42.4}{5.212} - \frac{10j}{2.12} = \frac{-\sqrt{2} \cdot 1.4 + j1.4\sqrt{2}}{2.12} - \frac{2.12 + 2.12j}{2.12}$   
 $U_6 = \frac{16-j56 + 8\sqrt{2} + j42.4}{5} - 10j = \frac{-4\sqrt{2} - 2.12 + j2.12 + j1.4\sqrt{2}}{5}$   
 $U_7 = (1.856 - 7.7j) = -2.17,86 + j23,18$   
 $U_8 = \frac{-2.17,86 + j23,18}{1.856 - 7.7j} = \frac{(-2.17,86 + j23,18)(1.856 + 7.7j)}{40.90 + 1.2} = -1,44 + j0,65$

$I_1 = \frac{U_6 - U_1}{1+j3.5} = \frac{-\frac{\sqrt{2}}{4} + 1,44 - j0,65}{1+j3.5} = \frac{(1,09 - j0,65)(1-j3,5)}{13,25} = -0,09 - j0,337$   
 $I_3 = -I_1 = 0,09 + j0,337$   
 $I_2 = I_3 - I_4 = -j \frac{\sqrt{2}}{2} + 0,09 + j0,337 = 0,09 - j0,337$

~~$U_1 = \frac{4-j4}{5} + \frac{4+j2}{5} - 0.5j = \frac{-\sqrt{2} \cdot 4 - j4\sqrt{2}}{2.12} + j(1+j)$~~   
 ~~$U_2 = \left( \frac{1-j3.5}{1+j3.5} + \frac{1+j0.5}{1-j0.5} - \frac{j}{2} \right) = \frac{-\sqrt{2}}{4+j4} + j(1+j)$~~   
 ~~$U_3 = \left( \frac{1-j3.5}{1.25} + \frac{1+j0.5}{1.25} - \frac{j}{2} \right) = \frac{-2 \cdot (4-j4)}{16+j16} + j(1+j)$~~

$I_4 = \frac{-U_1}{Z_x} = \frac{-1,44 - j0,65}{1-j0,5} = \frac{(1,44 + j0,65)(1+j0,5)}{1,25} = 1,142 + j0,056$   
 $I_5 = \frac{U_3 + U_6}{2j} = \frac{-1,44 + j0,65 + 2 + j2}{2j} = \frac{0,56 + j2,65}{2j} = 1,325 - j0,286$

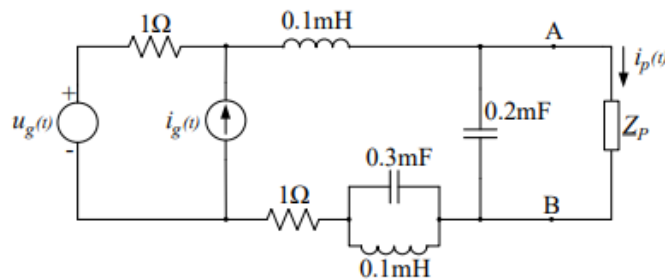
$S_1 = I_1^2 \cdot Z_x = (1,42 + j0,056)^2 \cdot (1 - j \frac{1}{2}) = (2 + 0,16j)(1 - j0,5)$   
 $S = 2,08 - j0,44$   
 $P = 2,08 \text{ W}$   
 $Q = -0,44 \text{ VAR}$

$I_1 = 0,35\sqrt{2} \cos(\omega t - 10,8^\circ)$   
 $I_2 = 0,38\sqrt{2} \cos(\omega t - 76,5^\circ)$   
 $I_3 = 0,35\sqrt{2} \cos(\omega t + 75,2^\circ)$   
 $I_4 = 2,44\sqrt{2} \cos(\omega t + 2,2^\circ)$   
 $I_5 = 1,35\sqrt{2} \cos(\omega t - 11,9^\circ)$

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2. U kolu sa slike uspostavljen je prostoperiodični režim i poznato je  $u_g(t) = 8\sqrt{2}V\cos(\omega t + 45^\circ)$  i  $i_g(t) = 8A\sin(\omega t)$  gde je  $\omega = 10\text{krad/s}$ . Potrebno je:

- [1] za deo kola levo od tačka A i B izračunati parametre Tevenenovog generatora u kompleksnom domenu;
- [4] za parametre izračunate pod tačkom a) izračunati ekvivalentni Tevenenov generator u vremenskom domenu;
- [2] odrediti impedansu  $Z_P$  tako da se na njoj razvija maksimalna aktivna snaga;
- [3] za impedansu izračunatu pod tačkom c) odrediti struju  $i_p(t)$  u vremenskom domenu.



$u_g(t) = 8\sqrt{2} \cos(\omega t + 45^\circ)$   
 $i_g(t) = 8A \sin \omega t$   
 $\omega = 10 \cdot 10^3 \text{ rad/s}$

$U_g = 8 \angle 45^\circ$   
 $\underline{I}_g = 8 \cdot (\cos 45^\circ + j \sin 45^\circ)$   
 $\underline{I}_g = 8 \cdot \frac{\sqrt{2}}{2} + j 4\sqrt{2}$   
 $\underline{I}_g = 4\sqrt{2} + j 4\sqrt{2}$   
 $\underline{I}_g = \frac{8}{\sqrt{2}} = 4\sqrt{2} \angle -90^\circ$   
 $\underline{I}_g = -j 4\sqrt{2}$

$X_{L01} = j \cdot \omega L = j \cdot 10 \cdot 10^3 \cdot 0,1 \cdot 10^{-3}$   
 $X_{L01} = j \Omega$   
 $X_{C02} = -j \frac{1}{\omega C} = -j \frac{1}{0,3 \cdot 10^{-3} \cdot 10 \cdot 10^3} = -j \frac{1}{3} \Omega$   
 $X_{C03} = -j \frac{1}{\omega C} = -j \frac{1}{0,2 \cdot 10^{-3} \cdot 10 \cdot 10^3} = -j \frac{1}{2} \Omega$   
 $X_{L04} = j \cdot \omega L = j \cdot 10 \cdot 10^3 \cdot 0,1 \cdot 10^{-3} = j \Omega$   
 $X_{C05} = -j \frac{1}{\omega C} = -j \frac{1}{0,3 \cdot 10^{-3} \cdot 10 \cdot 10^3} = -j \frac{1}{3} \Omega$

$\underline{U}_{AB} = \underline{I}_{AB} \cdot X_{C02} = \frac{\underline{U}_g}{1 + j\Omega - j\frac{1}{3}\Omega - j\frac{1}{2}\Omega + j\Omega + 1\Omega} \cdot (-j \cdot 0,15) = \frac{4\sqrt{2} + j 4\sqrt{2}}{2} = 2\sqrt{2} + j 2\sqrt{2}$

$\underline{U}_{AB} = \frac{1\Omega \cdot \underline{I}_g \cdot (-j 0,15)}{2\Omega} = -j 2\sqrt{2} \cdot (-j 0,15)$

$\underline{U}_{AB} = 2\sqrt{2} - (-j 0,15) = -j \sqrt{2} V$

$\underline{Z}_{TEV} = \frac{(1\Omega + 1\Omega + j\Omega - j\frac{1}{3}\Omega) \cdot (-j 0,15) \cdot (2 + 0,15j) \cdot (-j 0,15) \Omega}{2}$

$\underline{Z}_{TEV} = \frac{-j + 0,25}{2} = 0,125 - 0,15j = \frac{1}{8} - \frac{1}{2}j$

$\underline{U}_{AB} = \sqrt{2} \cdot (\cos \omega t + j \sin \omega t)$   
 $U_{AB} = 2 \cdot \cos(\omega t - 90^\circ)$   
 $u_{AB} = 2 \cdot \sin(\omega t)$   
 $R_{TEV} = \frac{1}{8} \Omega \quad C_{TEV} = 0,12 \mu F$   
 $\underline{Z}_0 = \underline{Z}_T^* = \frac{1}{8} + \frac{1}{2}j$

$I_P = \frac{U_{TEV}}{2 \cdot \frac{1}{8}} = \frac{j\sqrt{2}}{\frac{1}{4}} = j 4\sqrt{2}$   
 $i_p = 8A \sin \omega t$

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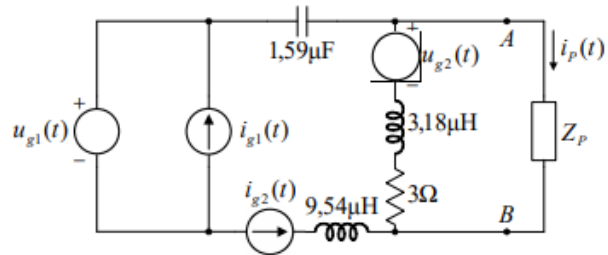
3. Kolo naizmenične struje sa slike radi u ustaljenom prostoperiodičnom režimu na frekvenciji  $f = 50\text{kHz}$ . Poznato je da je  $u_{g1}(t) = 2\text{V} \cos(2\pi ft + 45^\circ)$ ,  $u_{g2}(t) = -2\sqrt{2}\text{V} \sin(2\pi ft)$ ,  $i_{g1}(t) = 2\text{A} \cos(2\pi ft - 45^\circ)$  i  $i_{g2}(t) = -\sqrt{2}\text{A} \sin(2\pi ft)$ .

a) [60] Odrediti parametre ekvivalentnog Nortonovog generatora u kompleksnom domenu za deo kola levo od tačaka A i B.

b) [10] Odrediti elemente (i vrednosti elemenata) koji treba da sačinjavaju potrošač  $Z_p$ , tako da se na njemu razvija maksimalna aktivna snaga.

c) [20] Pod uslovom iz tačke b) odrediti kompleksnu, aktivnu i reaktivnu snagu na potrošaču  $Z_p$ .

d) [10] Pod uslovom iz tačke b) odrediti struju  $i_p(t)$ .



a)

$f = 50\text{kHz}$   
 $u_{g1} = 2\text{V} \cos(2\pi ft + 45^\circ)$   
 $u_{g2} = -2\sqrt{2}\text{V} \sin(2\pi ft) \stackrel{*32}{=} 2\sqrt{2}\text{V} \cos(2\pi ft + 90^\circ)$   
 $i_{g1} = 2\text{A} \cos(2\pi ft - 45^\circ)$   
 $i_{g2} = -\sqrt{2}\text{A} \sin(2\pi ft) \stackrel{*32}{=} \sqrt{2}\text{A} \cos(2\pi ft + 90^\circ)$

$U_{g1} = \frac{\sqrt{2}}{2} \cdot (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) = 0.5 + j0.5$   
 $U_{g2} = 2 \cdot (\cos 90^\circ + j\sin 90^\circ) = 2j$   
 $I_{g1} = \frac{\sqrt{2}}{2} (\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}) = 0.5 - j0.5$   
 $I_{g2} = 1 \cdot (\cos 90^\circ + j\sin 90^\circ) = j$

$X_C = \frac{-j}{10050 \cdot 10^3} = -j2 \cdot 10^{-8}$ ,  $X_{L1} = j \cdot 2\pi f L = j2$ ,  $X_{L2} = j32$

$U_{AB} = U_{AB}^I + U_{AB}^{II} + U_{AB}^{III} + U_{AB}^{IV} = 0\text{V} + U_{g2} + 0\text{V} + (-I_{g2}(X_{L1} + R_1)) = 2j - j(3+j) = 1-j$

$Z_N = Z_{TeV} = R + X_{L1} = (3+j) \cdot 10^{-2}$ ,  $I_N = \frac{U_{AB}}{Z_{TeV}} = \frac{1-j}{3+j} \cdot \frac{3-j}{3-j} = \frac{3-1-3j-j}{10} = (0.2 - 0.4j) \text{ A}$

b)  $Z_p = Z_N^* = 3-j$ ,  $R_p = 3 \cdot 10^{-2} \Omega$ ,  $C_p = 3.18 \mu\text{F}$

$I_p = \frac{1-j}{6 \cdot 10^{-2}} = (\frac{1}{6} - \frac{j}{6}) \text{ A}$   
 $|I_p| = \sqrt{\frac{1}{36} + \frac{1}{36}} = \frac{\sqrt{2}}{6}$   
 $|I_p|^2 = \frac{2}{36} = \frac{1}{18}$

$S = |I_p|^2 Z_p = \frac{1}{18} \cdot (3-j)$   
 $S = \frac{1}{6} - \frac{j}{18}$   
 $P = \frac{1}{6} \text{ W}$ ,  $Q = -\frac{1}{18} \text{ VAR}$

$i_p = \frac{\sqrt{2}}{6} (\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2})$   
 $i_p(t) = \sqrt{2} \cdot \frac{\sqrt{2}}{6} \cos(2\pi ft - 45^\circ)$   
 $i_p(t) = \frac{1}{3} \cos(100000\pi t - 45^\circ)$

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