NEODREĐENI INTEGRAL

Neka je $f(x)$ definisana u intervalu (a,b). Primitivna funkcija funkcije $f(x)$ na datom intervalu je bilo koja diferencijabilna funkcija $F(x)$ za koju je $F'(x) = f(x),$ za svako $x \in (a,b)$. Neodređeni integral funkcije $f(x)$ na intervalu (a,b) je skup svih primitivnih funkcija te funkcije i označava se sa

$$\int f(x)dx = F(x) + C.$$

Osnovna pravila:

$$d\left(\int f(x)dx\right) = f(x)dx$$

$$\int df(x) = f(x) + C$$

$$\int af(x)dx = a\int f(x)dx, \ (a \in R \setminus \{0\})$$

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx.$$

Teoreme o smeni promenljive:

1. $(x = \varphi(t))$

Ako su funkcije $f,$ φ i φ' neprekidne, tada je $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt + C.$

2. $(\varphi(x) = t)$

Ako su funkcije $f,$ $\varphi,$ φ^{-1} i $(\varphi^{-1})'$ neprekidne, tada je $\int f(\varphi(x))dx = \int f(t)(\varphi^{-1}(t))'dt + C.$

Tablica integrala

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x} dx = tgx + C$$

$$\int \frac{1}{\sin^2 x} dx = -ctgx + C$$

$$\int shx dx = chx + C$$

$$\int chx dx = shx + C$$

$$\int \frac{1}{ch^2 x} dx = thx + C$$

$$\int \frac{1}{sh^2 x} dx = -cthx + C$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} arctg \frac{x}{a} + C$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + C$$

1. Izračunati:

1) $\int \frac{x^3 - x^2}{2x} dx$, 2) $\int \frac{6 + 2x - x^2}{x^3} dx$, 3) $\int \sqrt{x} dx$, 4) $\int \frac{dx}{\sqrt{x}}$, 5) $\int \frac{x-1}{\sqrt[3]{x^2}} dx$,

6) $\int \frac{e^{2x} - 1}{e^x} dx$, 7) $\int \frac{2^x + 3^x}{6^x} dx$, 8) $\int \frac{dx}{\sin^2 x \cos^2 x}$, 9) $\int tg^2 x dx$, 10) $\int (\sin \frac{x}{2} - \cos \frac{x}{2}) dx$,

11) $\int \frac{x^2}{3 + x^2} dx$, 12) $\int \frac{x^3 - 4x + 1}{x^2 - 4} dx$, 13) $\int \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1-x^4}} dx$, 14) $\int \frac{1 + 2x^2}{x^2(1+x^2)} dx$, 15) $\int \frac{x^4}{1-x^2} dx$.

2. Izračunati, uvođenjem pogodne smene:

1) $\int (2x-5)^{11} dx$, 2) $\int \sqrt[3]{2-3x} dx$, 3) $\int e^{-x} dx$, 4) $\int \sin 5x dx$, 5) $\int \frac{x}{1+x^2} dx$,

6) $\int \frac{2x-3}{x^2 - 3x + 4} dx$, 7) $\int tgx dx$, 8) $\int \frac{\sin x}{2 + \cos x} dx$, 9) $\int \frac{dx}{x \ln x}$, 10) $\int \frac{e^x}{e^{2x} - 1} dx$,

11) $\int \frac{dx}{\sin x}$, 12) $\frac{dx}{\cos x}$, 13) $\int x^2 \sqrt[3]{1+x^3} dx$, 14) $\int \frac{x^2 dx}{x^6 + 3}$, 15) $\int \frac{dx}{(1+x)\sqrt{x}}$,

16) $\int \frac{1}{x^2} \sin \frac{1}{x} dx$, 17) $\int \cos^3 x \sin x dx$, 18) $\int \frac{dx}{\sin^4 x}$, 19) $\int \frac{arctgx}{1+x^2} dx$, 20) $\int \frac{x^2 dx}{(1-x)^{100}}$.

3. Izračunati:

1) $\int \ln x dx$, 2) $\int arctgx dx$, 3) $\int x\cos 3x dx$, 4) $\int (x^2-2x+5)e^{-x}dx$,

5) $\int x^2 e^{3x}dx$, 6) $\int x\sin x\cos x dx$, 7) $\int \frac{xdx}{\cos^2 x}$,

8) $\int \frac{\ln x dx}{\sqrt{x}}$, 9) $\int \ln(x+\sqrt{x^2+1})dx$, 10) $\int x\arcsin x dx$.

4. Izračunati:

1) $\int e^{ax}\sin bx dx$, $(a,b\in R)$ 2) $\int \sin(\ln x)dx$, 3) $\int \sqrt{a^2-x^2}dx$,

4) $\int \sqrt{a^2+x^2}dx$, 5) $\int \sqrt{x^2-a^2}dx$, 6) $\int \frac{x^2 dx}{(x^2+1)^2}$.

5. Izračunati:

1) $\int e^{\sqrt{x}}dx$, 2) $\int \frac{x^2 arctgx dx}{x^2+1}$, 3) $\int \arcsin^2 x dx$, 4) $\int \frac{\ln(\ln x)}{x}dx$.

6. Izračunati:

1) $\int \frac{dx}{x^2+2x+5}$, 2) $\int \frac{dx}{3x^2-x+1}$, 3) $\int \frac{dx}{\sqrt{2+3x-2x^2}}$, 4) $\int \frac{xdx}{x^2-7x+13}$,

5) $\int \frac{2x-6}{\sqrt{x^2-4x+5}}dx$, 6) $\int \sqrt{x^2-2x+5}dx$, 7) $\int x\sqrt{x^4-4x^2+3}dx$,

8) $\int \frac{\cos x dx}{\sin^2 x-6\sin x+12}$, 9) $\int \frac{e^x dx}{\sqrt{1+e^x+e^{2x}}}$, 10) $\int \frac{\ln x dx}{x\sqrt{1-4\ln x-\ln^2 x}}$.

7. Izračunati, koristeći metod neodređenih koeficijenata:

1) $\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}dx$, 2) $\int \frac{dx}{x^3-2x^2+x}$, 3) $\int \frac{x^2-8x+7}{(x^2-3x-10)^2}dx$,

4) $\int \frac{x^3+x+1}{x(x^2+1)}dx$, 5) $\int \frac{xdx}{x^3-1}$, 6) $\int \frac{dx}{x^4-1}$, 7) $\int \frac{dx}{x^4+1}$,

8) $\int \frac{x^4 dx}{x^4+5x^2+4}$, 9) $\frac{dx}{(1+x^2)^2}$, 10) $\int \frac{3x+5}{(x^2+2x+2)^2}dx$.

ODREĐENI INTEGRAL

Njutn-Lajbnicova formula:

$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$, ($F(x)$ je proizvoljna primitivna funkcija funkcije $f(x)$ na intervalu (a,b))

Neka je funkcija $f(x)$ neprekidna na $[-a,a]$. Tada:

$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$, ako je $f(x)$ parna, a $\int_{-a}^{a} f(x)dx = 0$, ako je $f(x)$ neparna.

1. Izračunati:

1) $\int_{-1}^{1} \sqrt[3]{x}\,dx$, 2) $\int_{0}^{\pi/2} \sin^2 x\,dx$, 3) $\int_{0}^{2} |1-x|\,dx$,

4) $\int_{0}^{2} f(x)dx$, gde je $f(x)=\begin{cases} x^2, & 0\le x\le 1 \\ 2-x, & 1<x\le 2 \end{cases}$.

2. Izračunati:

1) $\int_{-1}^{1} e^{x^2}\sin x\cos^4 x\,dx$, 2) $\int_{-\pi/4}^{\pi/4} \frac{x^3+x+1}{\cos^2 x}\,dx$, 3) $\int_{0}^{1} x(2-x)^{12}\,dx$,

4) $\int_{-1}^{1} \frac{x}{x^2+x+1}\,dx$, 5) $\int_{0}^{1} \frac{dx}{x^3+1}$, 6) $\int_{0}^{\pi/2} e^x \sin x\,dx$, 7) $\int_{0}^{1} 2x\,arctg\sqrt{x}\,dx$.