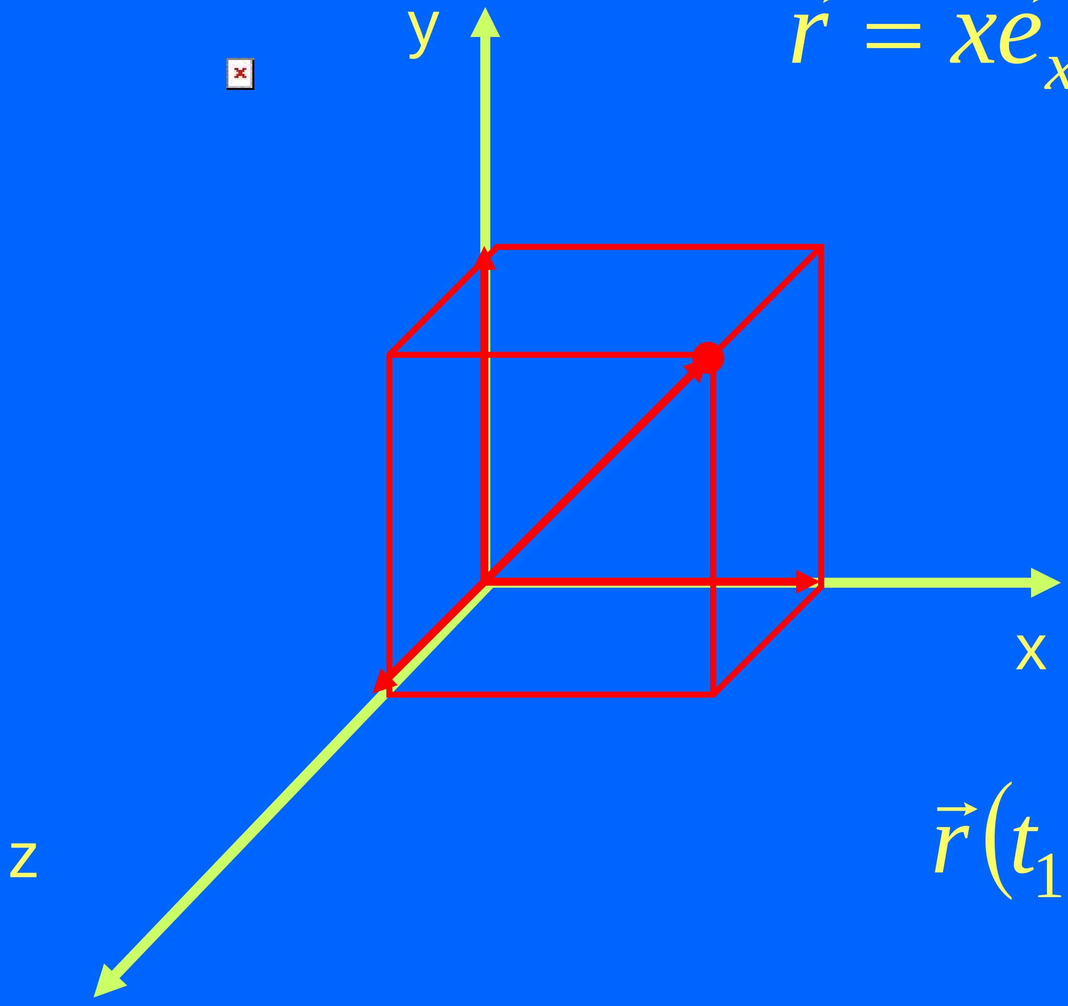
Kinematika

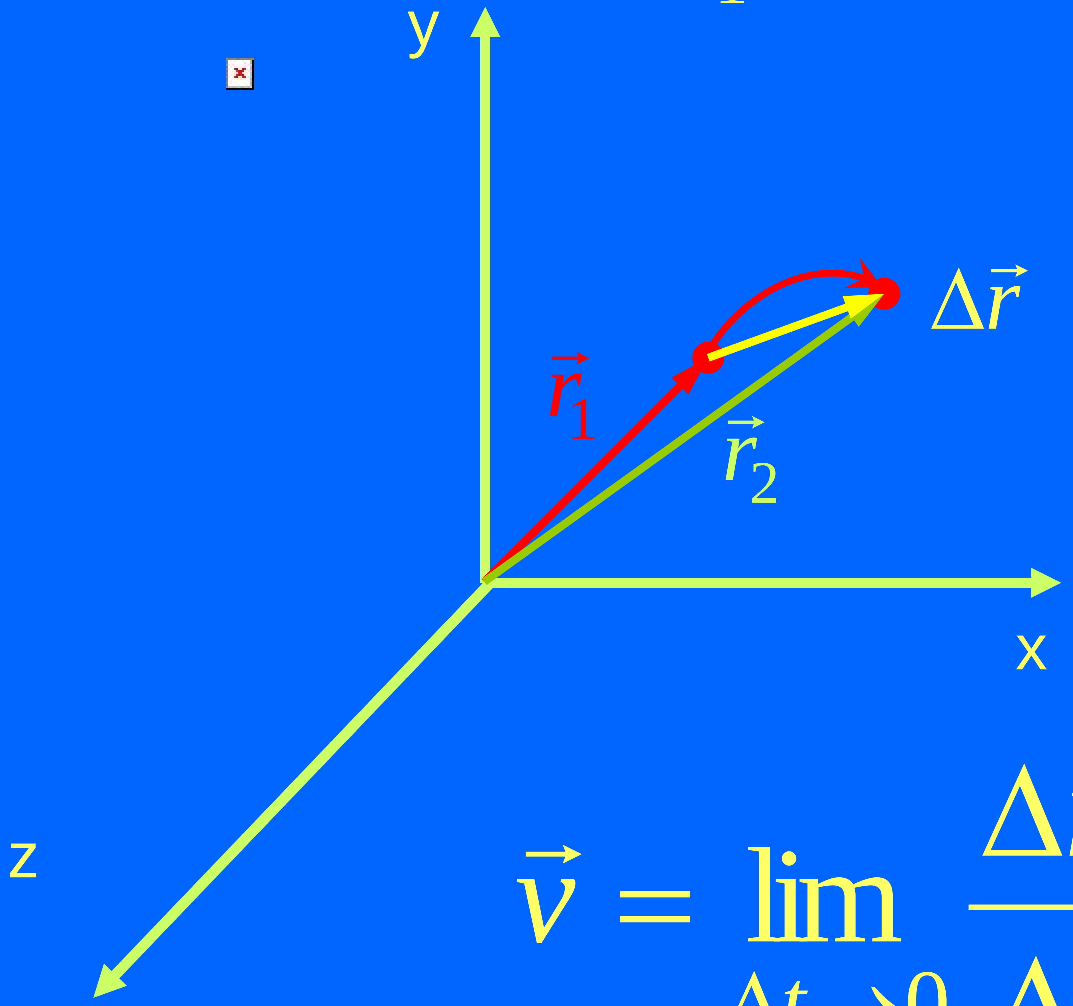
$$\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$$



$$\vec{r}(t_1) = \vec{r}_1$$

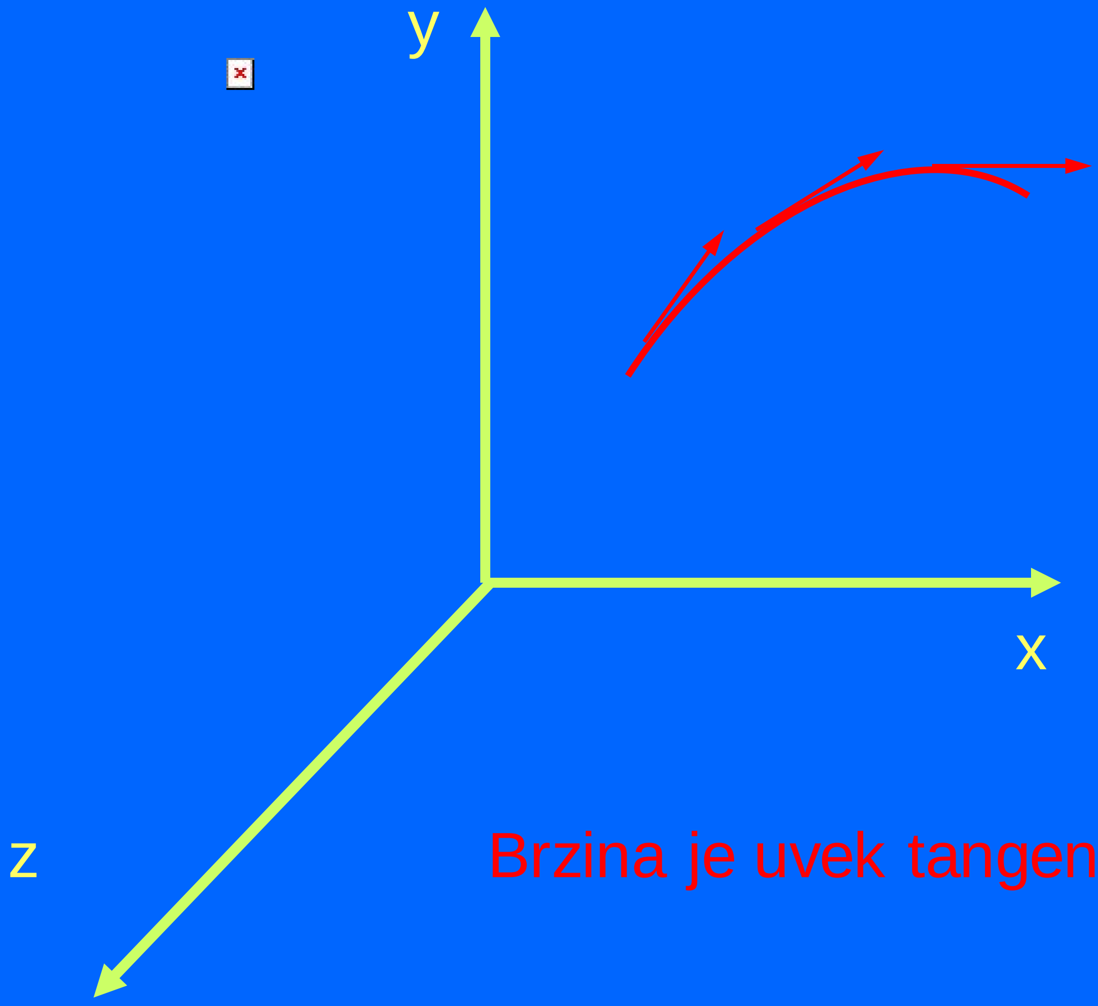
$$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2 \Rightarrow \Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta t = t_2 - t_1$$



$$\vec{v}_{sr} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} \equiv \frac{d\vec{r}}{dt}$$



Brzina je uvek tangentna na trajektoriju.

$$\vec{a}_{sr} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} \equiv \frac{d\vec{v}}{dt}$$

$$\vec{e}_x, \vec{e}_y, \vec{e}_z = const(t)$$

$$\vec{v} = v_x\vec{e}_x + v_y\vec{e}_y + v_z\vec{e}_z = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y + \frac{dz}{dt}\vec{e}_z$$

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt}$$

$$a_x = \frac{dv_x}{dt}, a_y = \frac{dv_y}{dt}, a_z = \frac{dv_z}{dt}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 \Rightarrow |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$a^2 = a_x^2 + a_y^2 + a_z^2 \Rightarrow |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{r} = \int \vec{v}dt + \vec{r}_0$$

$$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{v} = \int \vec{a}dt + \vec{v}_0$$

$$x = \int v_x dt + x_0 \qquad y = \int v_y dt + y_0 \qquad z = \int v_z dt + z_0$$

Ravnomerno pravolinijsko kretanje:

X osu kordinatnog sistema postavimo po pravcu kretanja sa koordinatnim početkom na mestu početnog položaja.

$$x = s, v_x = v, x = \int v_x dt + 0 = v_x t \Rightarrow s = vt \Rightarrow v = s / t$$

Ovo je jedini slučaj kada je v=s/t !!!

Pravolinijsko kretanje sa konstantnim ubrzanjem:

X osu kordinatnog sistema postavimo po pravcu ubrzanja sa koordinatnim početkom na mestu početnog položaja. Da bi pretpostavka važila početna brzina mora biti u pravcu ubrzanja.

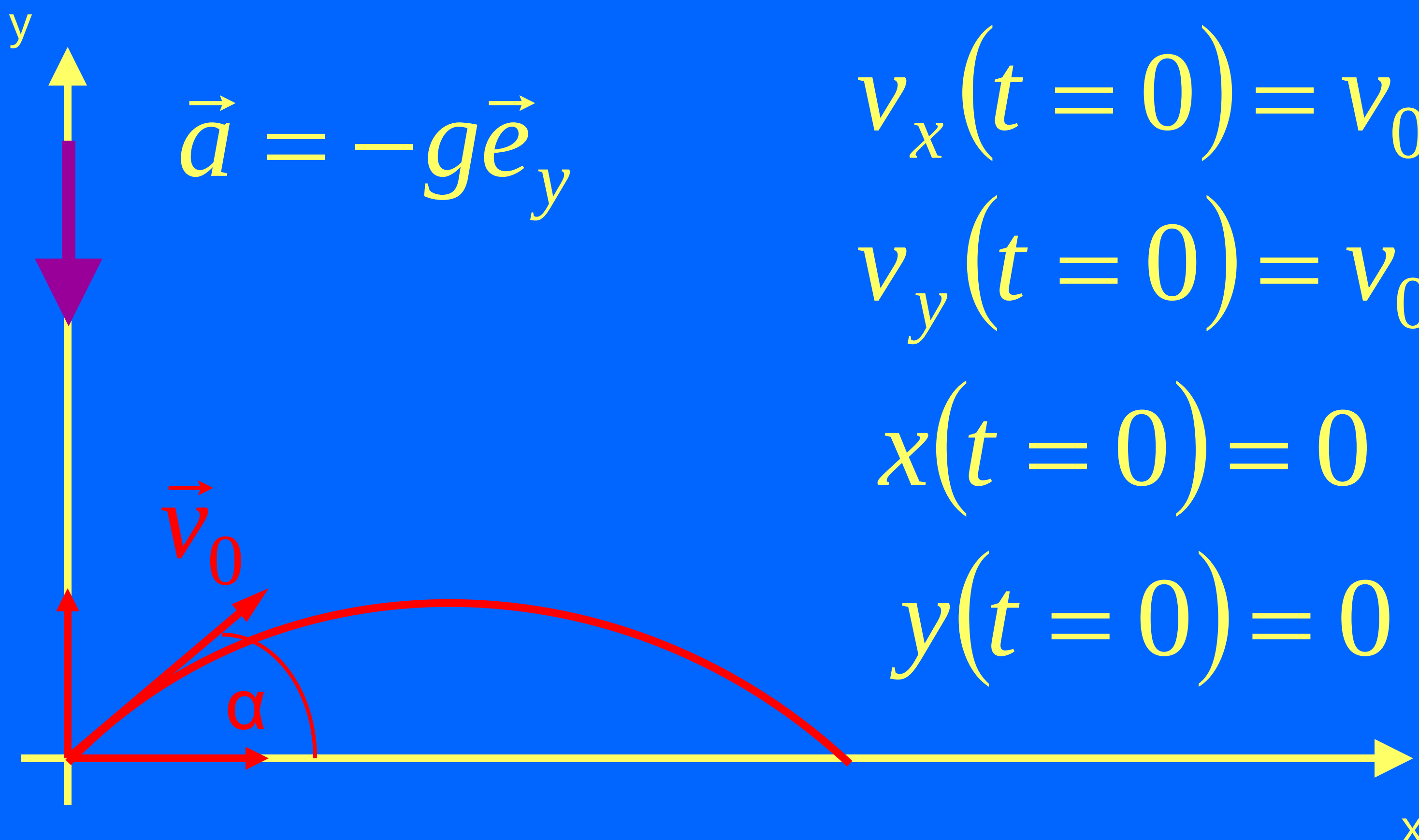
$$v_x = \int a_x dt + v_{x0} = a_x t + v_{x0}$$

$$x = \int v_x dt + x_0 = \int \left(a_x t + v_{x0}\right) dt = a_x \int t dt + v_{x0} \int dt$$

$$x = a_x \frac{t^2}{2} + v_{x0} t$$

Kosi hitac

$$\vec{a} = -g\vec{e}_y$$



$$v_x(t=0) = v_0 \cos\alpha$$

$$v_y(t=0) = v_0 \sin\alpha$$

$$x(t=0) = 0$$

$$y(t=0) = 0$$

$$a_x = 0 \Rightarrow v_x(t) = const. = v_x(t=0) = v_0 \cos\alpha$$

$$a_y = -g \Rightarrow v_y(t) = \int (-g)\,dt$$

$$v_y(t) = -gt + const.$$

$$v_y(t=0) = const. = v_0 \sin\alpha$$

$$v_y(t) = v_0 \sin\alpha - gt$$

$$x(t) = \int v_x\,dt \qquad y(t) = \int v_y\,dt$$

$$x(t) = v_0 t \cos\alpha$$

$$y(t) = v_0 t \sin\alpha - \frac{gt^2}{2}$$

Parametarske jednačine kretanja

Trajektorija:

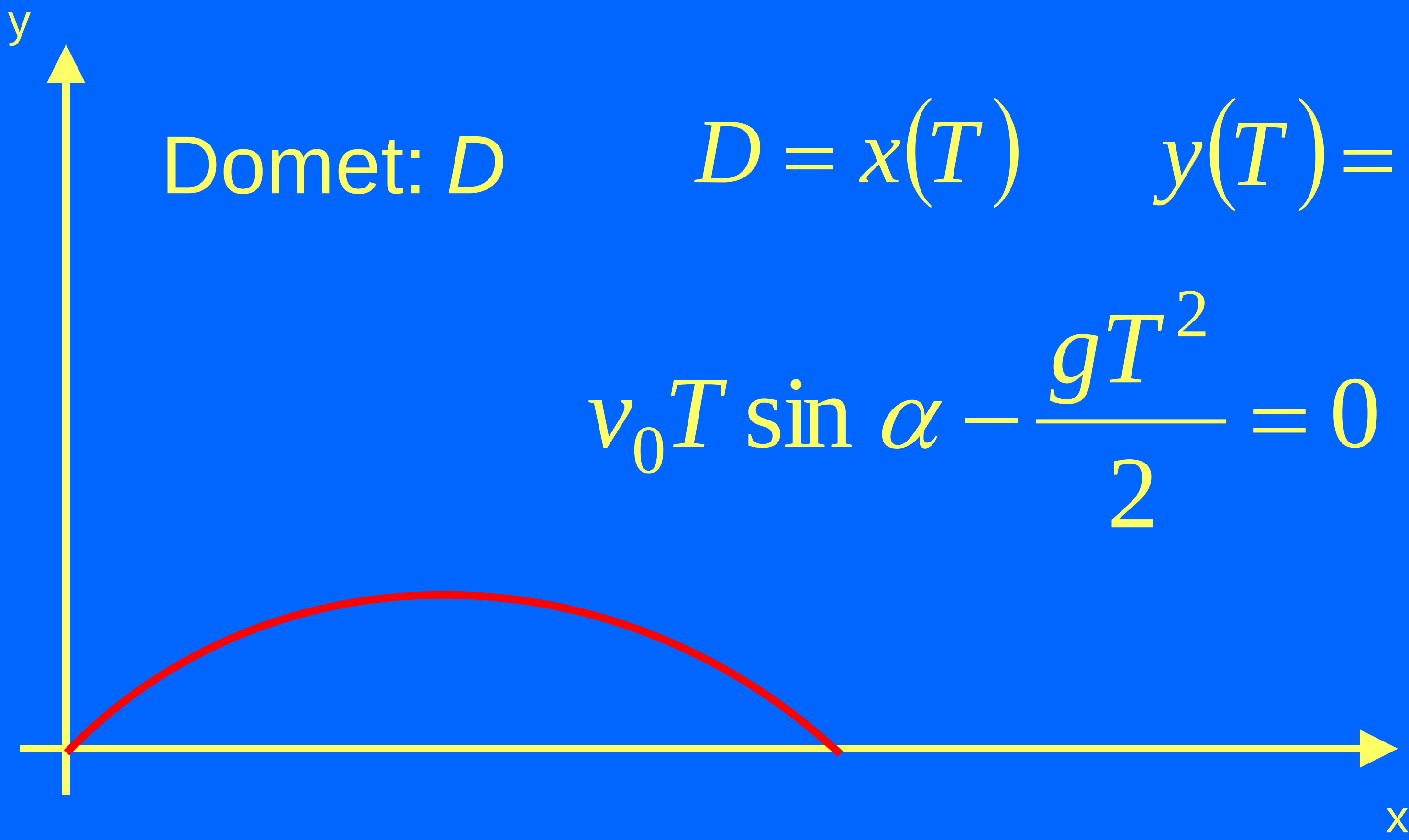
$$t = \frac{x}{v_0 \cos\alpha}$$

$$y(x) = x tg\alpha - x^2 \frac{g}{2v_0^2 \cos^2\alpha}$$

Domet: *D*

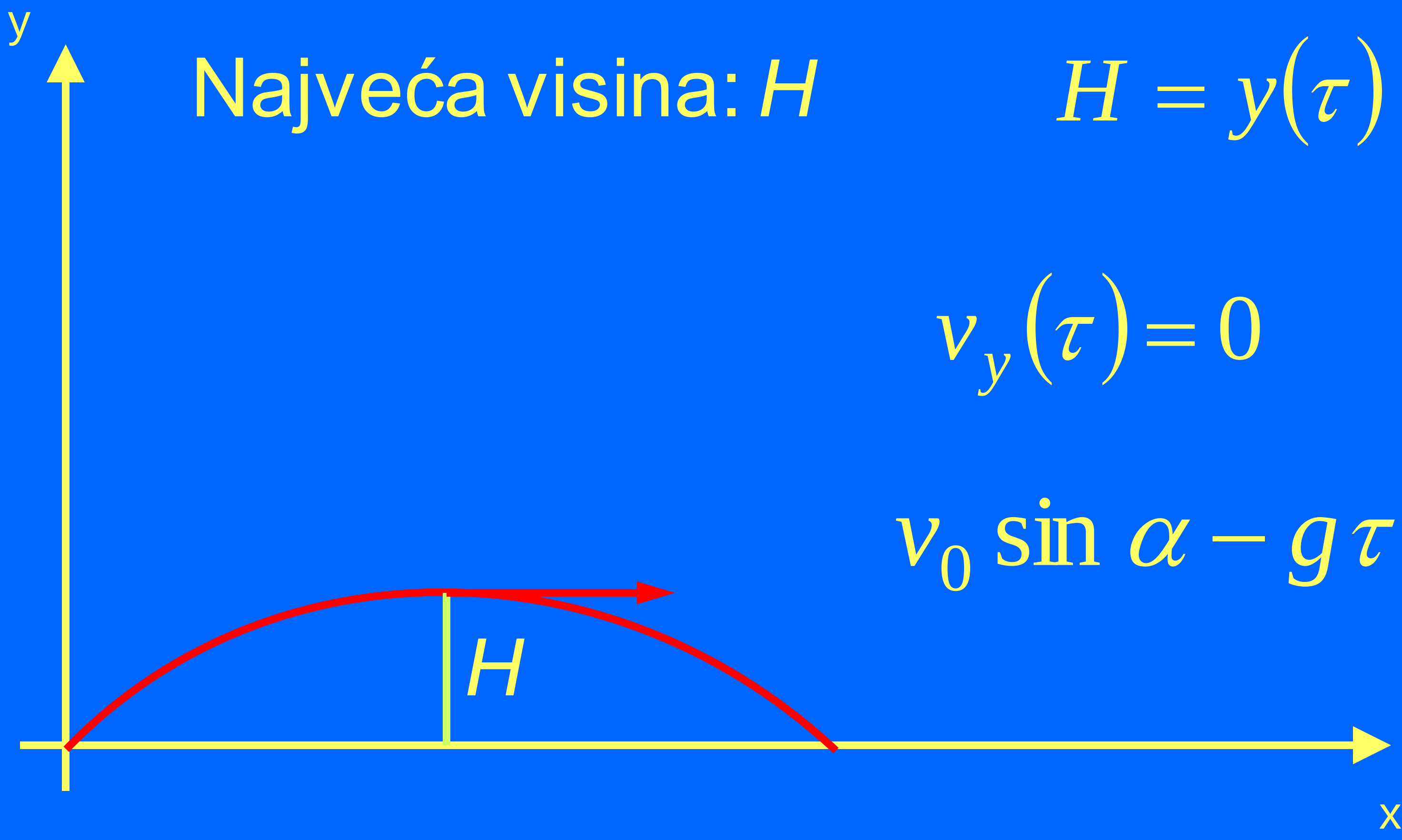
$$D = x(T) \qquad y(T) = 0$$

$$v_0 T \sin\alpha - \frac{gT^2}{2} = 0$$



$$T = \frac{2v_0 \sin\alpha}{g} \Rightarrow D = \frac{v_0^2}{g}\sin(2\alpha)$$

Najveća visina: *H*



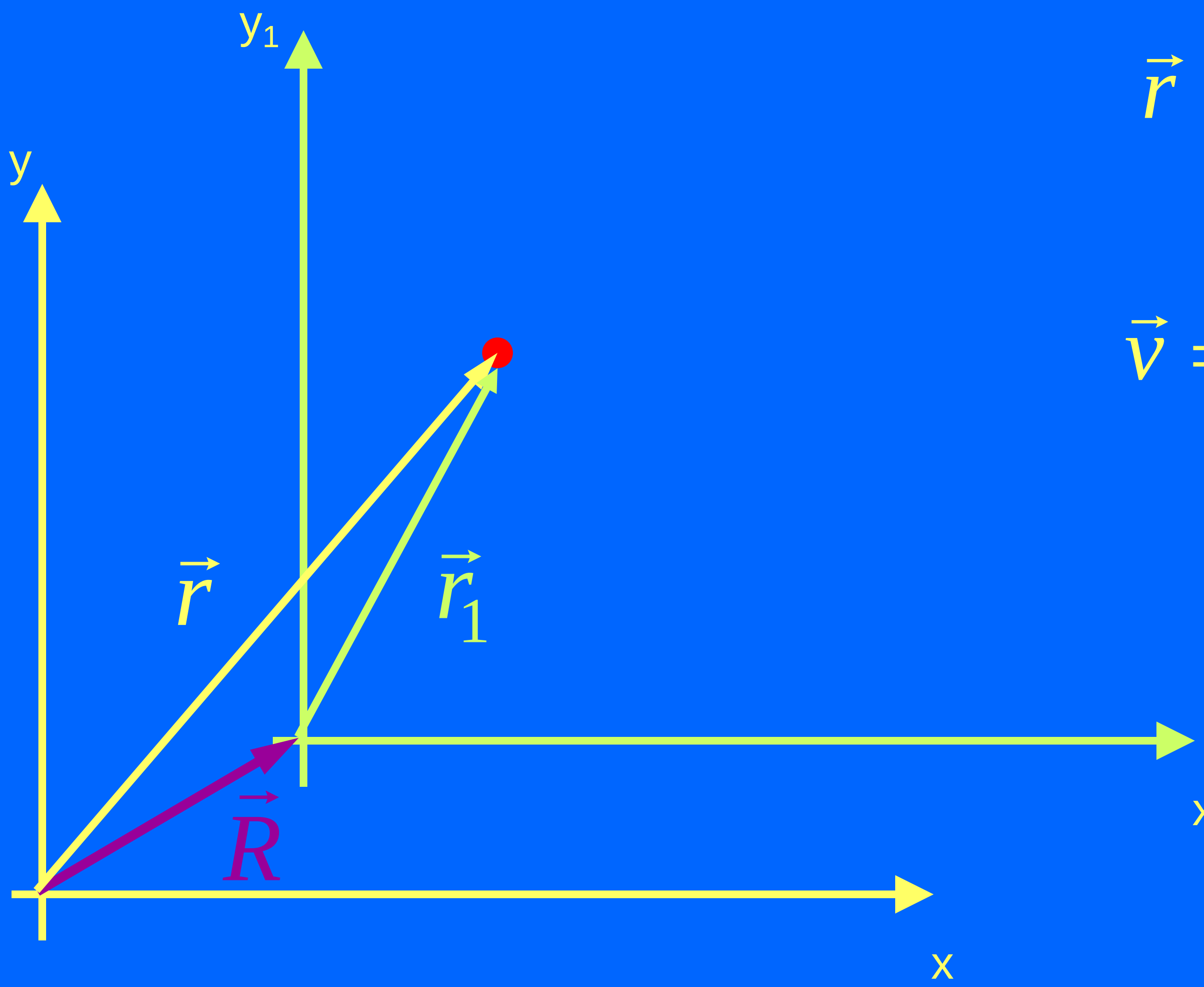
$$H = y(\tau)$$

$$v_y(\tau) = 0$$

$$v_0 \sin\alpha - g\tau = 0$$

$$\tau = \frac{v_0 \sin\alpha}{g} \Rightarrow H = \frac{v_0^2}{2g}\sin^2\alpha$$

Relativno kretanje

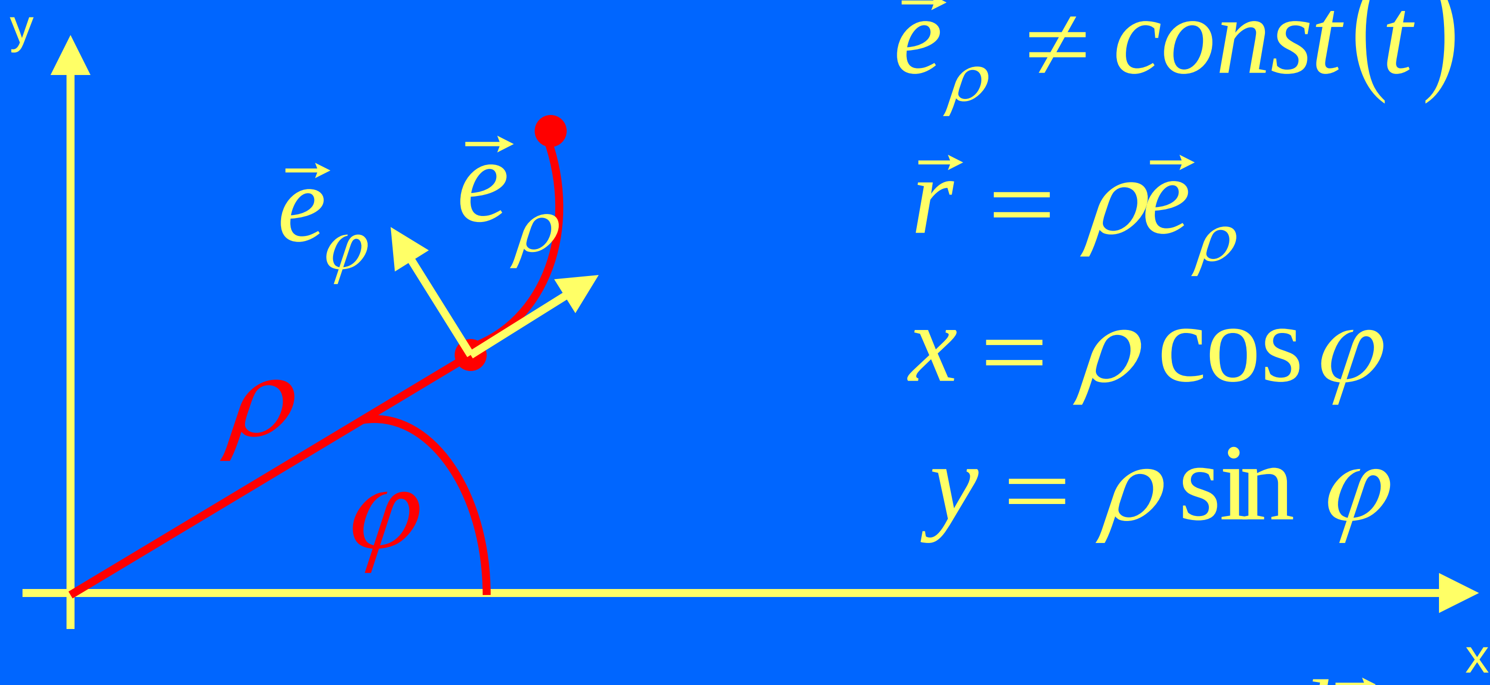


$$\vec{r} = \vec{R} + \vec{r}_1 \left| \frac{d}{dt} \right.$$

$$\vec{v} = \vec{V} + \vec{v}_1 \left| \frac{d}{dt} \right.$$

$$\vec{a} = \vec{A} + \vec{a}_1$$

Polarni koordinatni sistem



$$\vec{e}_\rho \neq const(t)$$

$$\vec{r} = \rho\vec{e}_\rho$$

$$x = \rho\cos\varphi$$

$$y = \rho\sin\varphi$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\rho\vec{e}_\rho\right) = \frac{d\rho}{dt}\vec{e}_\rho + \rho\frac{d\vec{e}_\rho}{dt}$$

$$\frac{d\vec{e}_\rho}{dt} = \frac{d\varphi}{dt}\vec{e}_\varphi \qquad \frac{d\vec{e}_\varphi}{dt} = -\frac{d\varphi}{dt}\vec{e}_\rho$$

Polarni koordinatni sistem

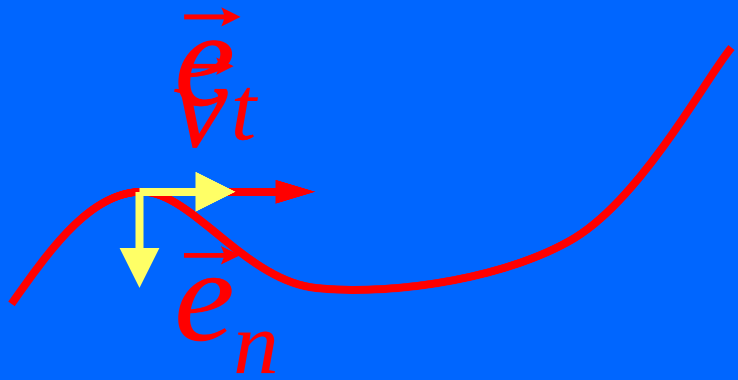
$$\vec{v} = \frac{d\rho}{dt}\vec{e}_\rho + \rho\frac{d\varphi}{dt}\vec{e}_\varphi \qquad v_\rho = \frac{d\rho}{dt} \qquad v_\varphi = \rho\frac{d\varphi}{dt}$$

$$v^2 = v_\rho^2 + v_\varphi^2 \Rightarrow \left|\vec{v}\right| = \sqrt{v_\rho^2 + v_\varphi^2}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\rho}{dt}\vec{e}_\rho + \rho\frac{d\varphi}{dt}\vec{e}_\varphi\right)$$

$$\vec{a} = \left(\frac{d^2\rho}{dt^2} - \rho\left(\frac{d\varphi}{dt}\right)^2\right)\vec{e}_\rho + \left(2\frac{d\rho}{dt}\frac{d\varphi}{dt}\rho\frac{d^2\varphi}{dt^2}\right)\vec{e}_\varphi$$

Određivanje položaja na prirodan način

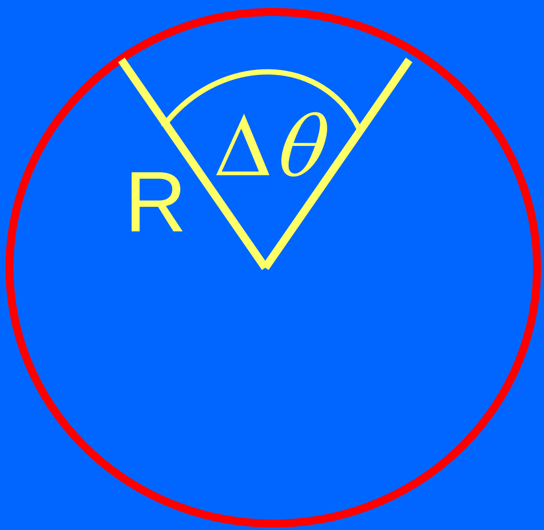


$$\vec v = v\vec e_t = \frac{ds}{dt}\vec e_t$$

$$\vec a = \frac{d\vec v}{dt} = \frac{dv}{dt}\vec e_t + v\frac{d\vec e_t}{dt} = \frac{dv}{dt}\vec e + \frac{v^2}{R}\vec e_n$$

$$a_\tau = \frac{dv}{dt} \qquad a_n = \frac{v^2}{R} \qquad S(\Delta t) = \int_{t1}^{t2} \left|\vec v\right| dt$$

Rotaciono kretanje



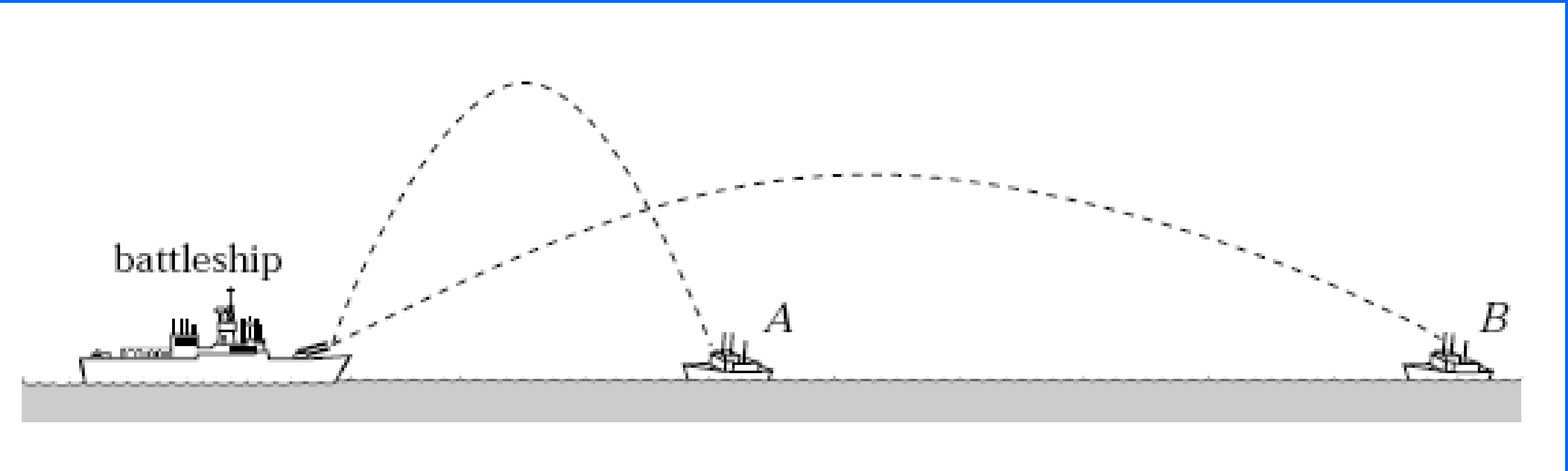
$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$ds = Rd\theta \Rightarrow v = R\omega$$

$$a_\tau = \frac{dv}{dt} \Rightarrow a_\tau = R\alpha$$

$$a_n = \frac{v^2}{R} = \omega^2 R$$

Ratni brod istovremeno puca na dva neprijateljska broda. Ako se granate kreću putanjama kosog hitca prvo će biti pogođen brod:



A

B

biće istovremeno pogođeni

nemoguće odrediti