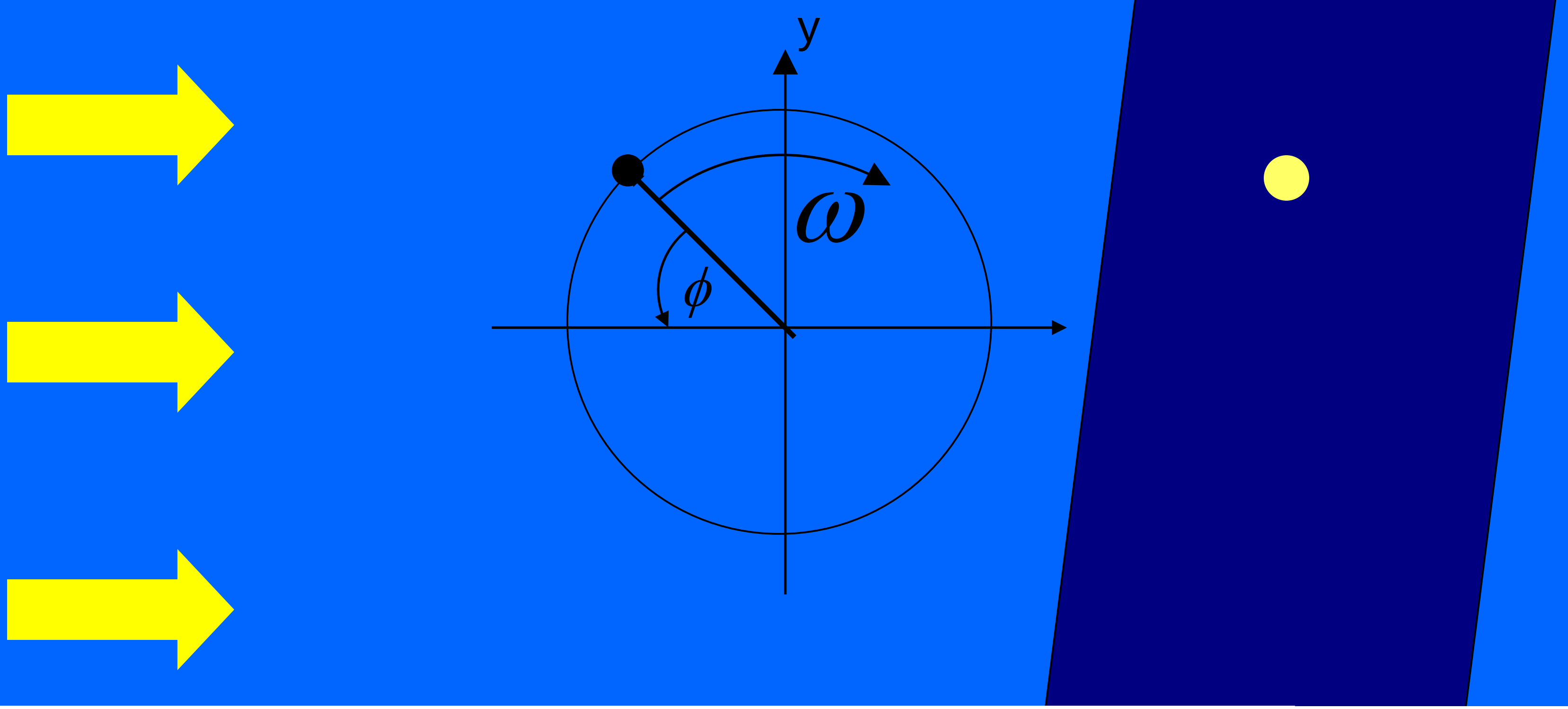
Oscilacije

U opštem slučaju ugaona brzina nije identična sa kružnom frekvencijom!



$$x(t) = A\cos(\omega t + \phi)$$

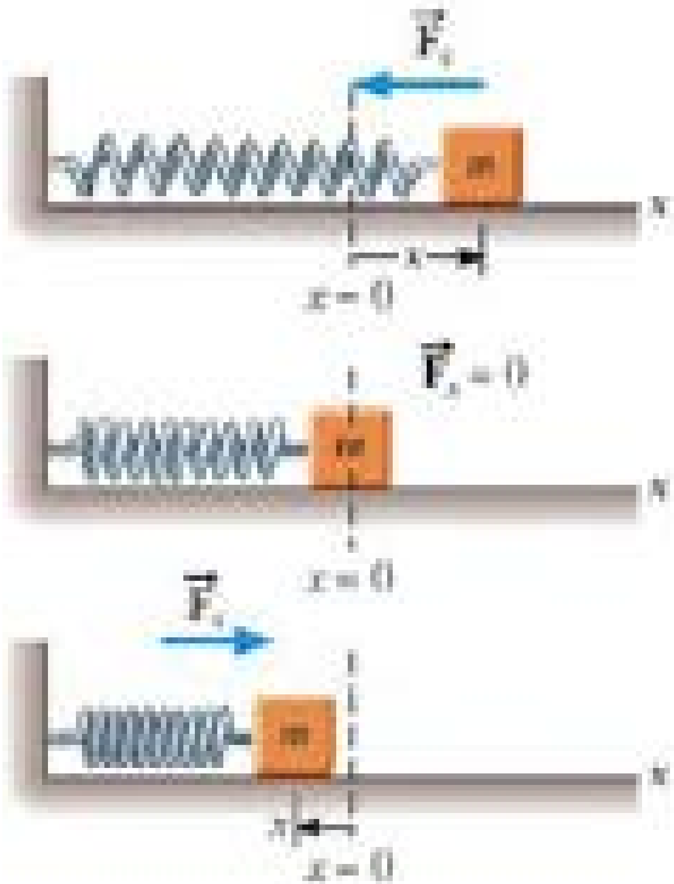
$$v(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi)$$

$$a(t) = \frac{dv(t)}{dt} = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x(t)$$

$A\omega = v_{max}$

$A\omega^2 = a_{max}$

Sistem teg opruga



$$F = -kx \quad F = ma \Rightarrow \quad ma = -kx$$

$$m\frac{\partial^2 x}{\partial t^2} = -kx$$

$$\frac{\partial^2 x}{\partial t^2} + \frac{k}{m}x = 0$$

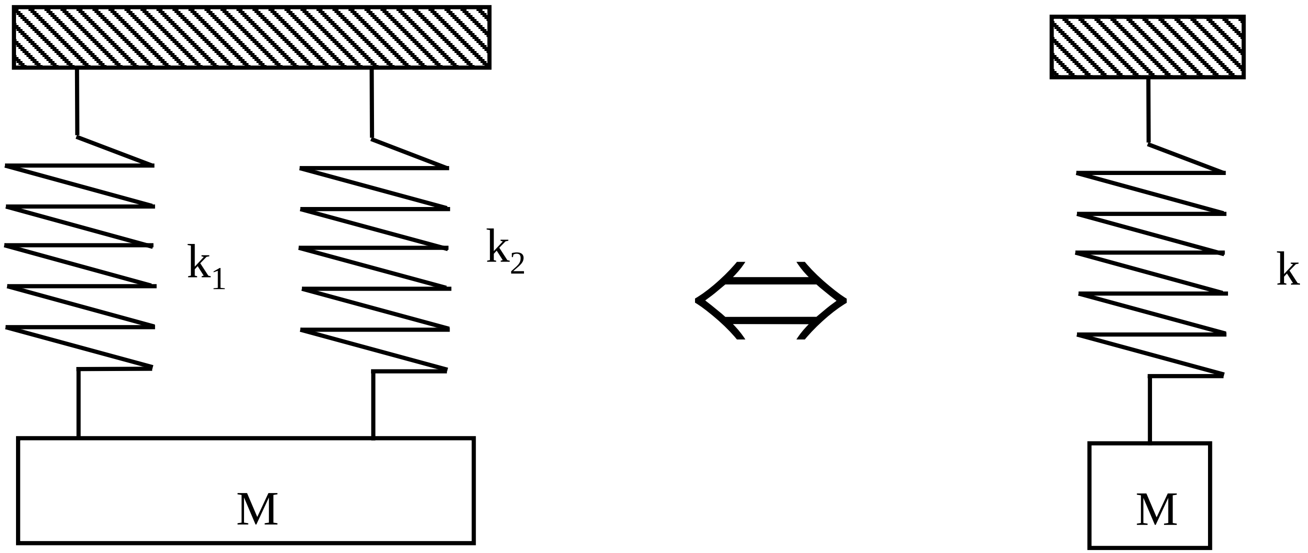
$$\frac{\partial^2 x}{\partial t^2} + \omega^2 x = 0 \qquad \omega^2 = \frac{k}{m}$$

$$x(t) = A\cos(\omega t + \phi)$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Amplituda ne utiče na frekvenciju!

Paralelna veza dve opruge



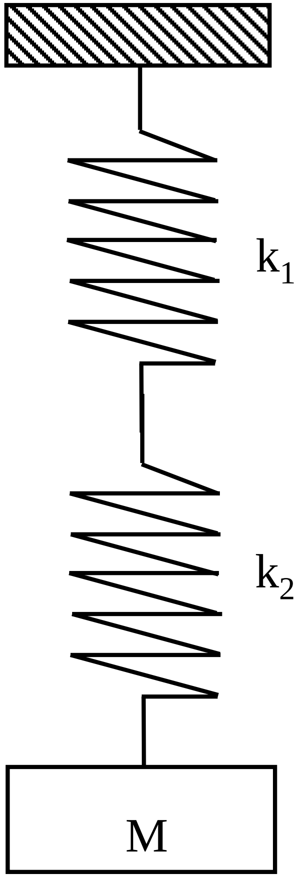
$$F = k_1 \Delta x + k_2 \Delta x$$

$$F = k_E \Delta x$$

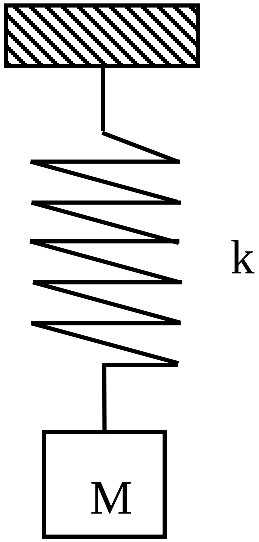
$$k_E = k_1 + k_2$$

Jednak pomeraj!

Redna veza dve opruge



$$\Leftrightarrow$$



$$\Delta x = \frac{F}{k_E} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$F = k_E \Delta x$$

$$k_E = \frac{k_1 k_2}{k_1 + k_2}$$

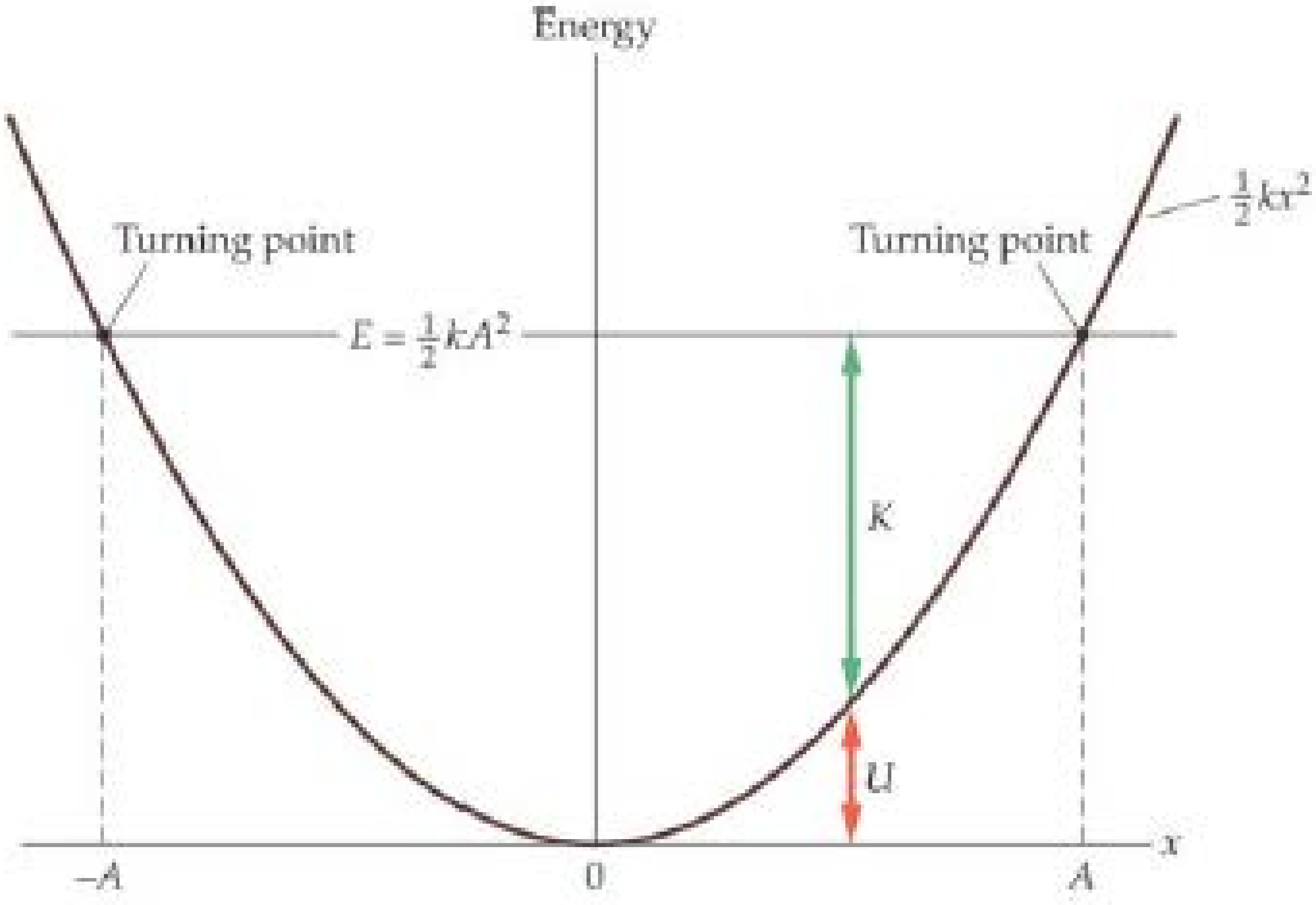
$$\Delta x = \Delta x_1 + \Delta x_2$$

$$F = k_1 \Delta x_1 = k_2 \Delta x_2$$

Jednaka sila!

Energija linearnog harmonijskog oscilatora

$\mathbf{E = U + K}$ $U = ½\,k\,x^2$ $K = ½\,m\,v^2$



Totalna energija sistema

$E = U_{max} + 0$ $\qquad$ $E = K_{max} + 0$

$E = U + K = \frac{1}{2} k (A \cos(\omega t))^2 + \frac{1}{2} m (A \omega \sin(\omega t))^2$

$= \frac{1}{2} k A^2 \cos^2(\omega t) + \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t)$

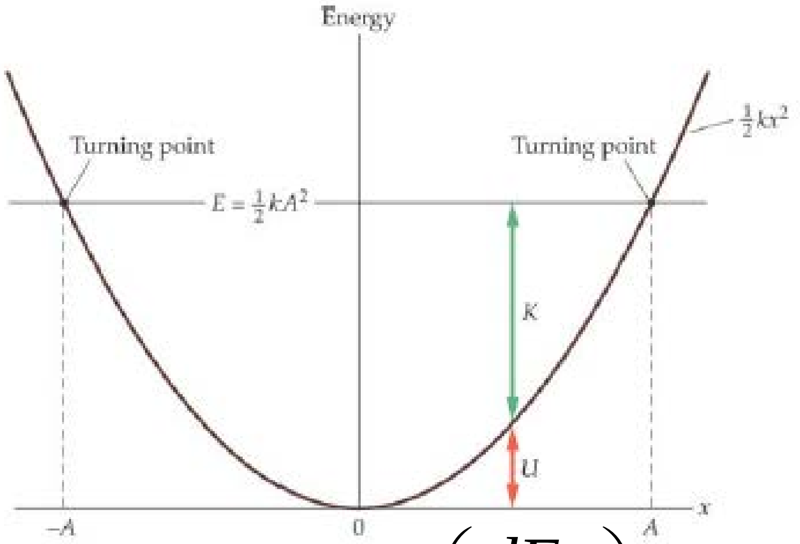
$E = \frac{1}{2} k A^2 \cos^2(\omega t) + \frac{1}{2} m A^2 \mathbf{k/m} \sin^2(\omega t) = \frac{1}{2} k A^2 (\cos^2(\omega t) + \sin^2(\omega t))$

$$E = \frac{1}{2} k A^2 = \frac{1}{2} m (A\omega)^2$$

Oscilacije su neprigušene

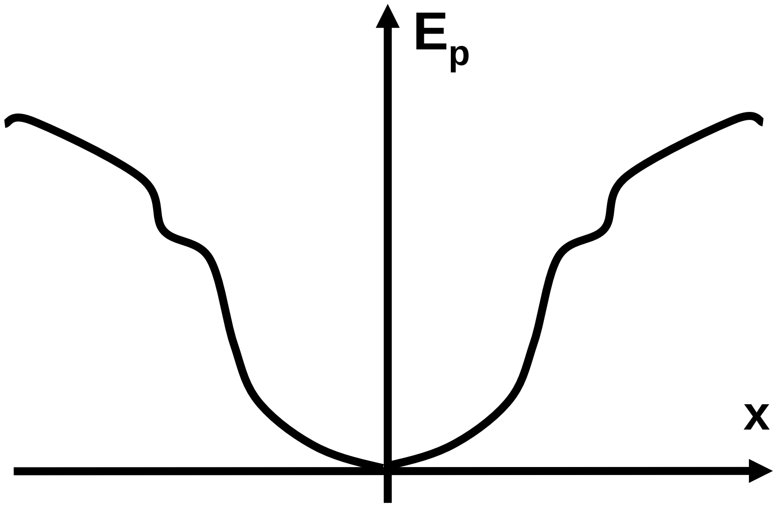
(smatra se da nema otporne sile) !

Linearni harmonijski oscilator



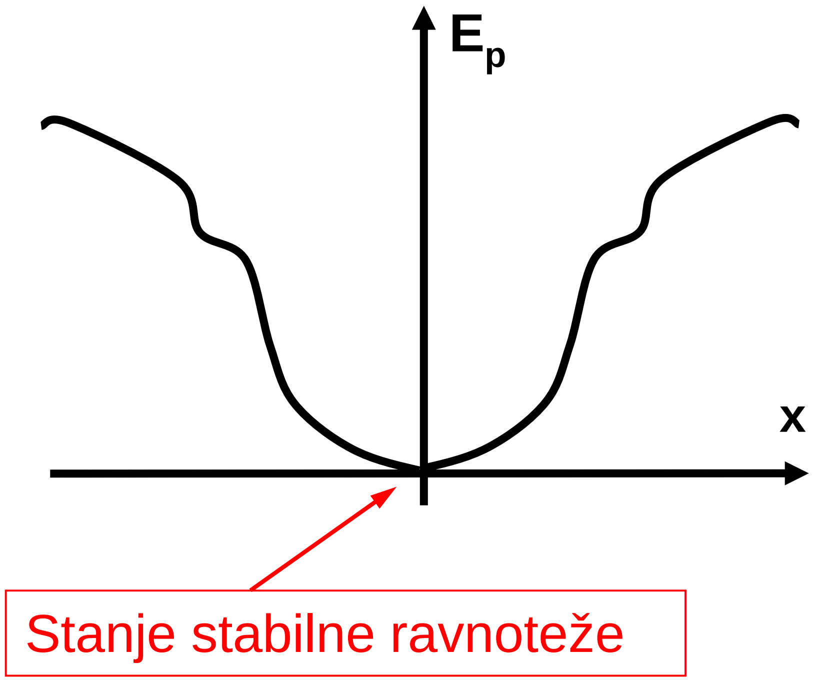
$$E_p = \frac{1}{2}kx^2$$

$$E_p(x) = E_p(0) + \left(\frac{dE_p}{dx}\right)_{x=0} x + \frac{1}{2!}\left(\frac{d^2E_p}{dx^2}\right)_{x=0} x^2 + \frac{1}{3!}\left(\frac{d^3E_p}{dx^3}\right)_{x=0} x^3 + \ldots$$



$$\left(\frac{dE_p}{dx}\right)_{x=0} = 0$$

Linearni harmonijski oscilator



$$x = \pm\varepsilon \qquad \varepsilon \to 0$$

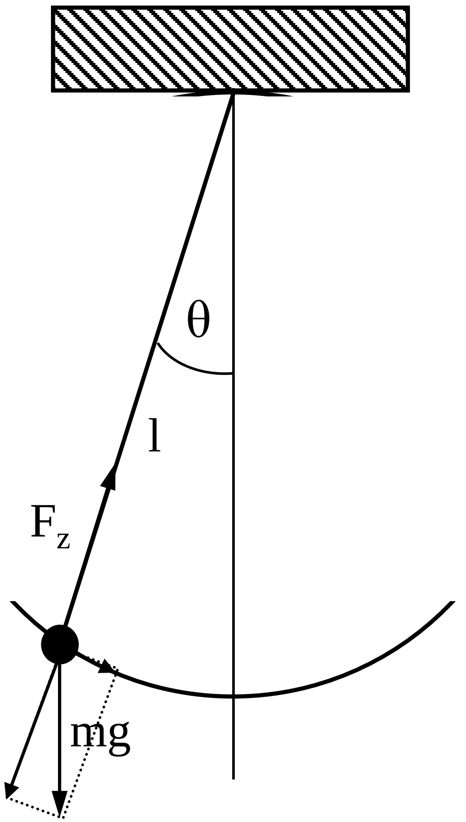
$$E_p(x) \approx E_p(0) + \frac{1}{2!}\left(\frac{d^2E_p}{dx^2}\right)_{x=0} x^2$$

$$\vec{F} = -gradE_p = -\left(\frac{d^2E_p}{dx^2}\right)_{x=0} x\vec{e}_x$$

$$\left(\frac{d^2E_p}{dx^2}\right)_{x=0} = const. = k$$

$$m\frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

Matematičko klatno



$$F_n = ma_n = F_z - mg\cos\theta$$

$$F_\tau = ma_\tau = -mg\sin\theta$$

$$a_\tau = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

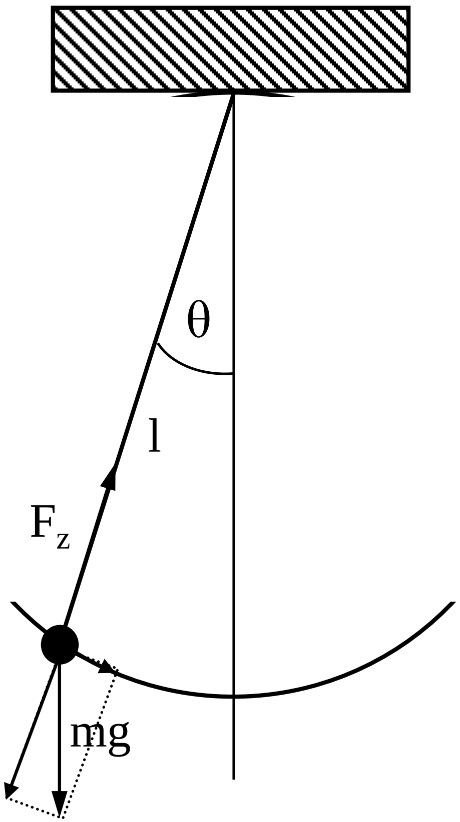
$$s = l\theta \Rightarrow \frac{d^2 s}{dt^2} = l\frac{d^2\theta}{dt^2}$$

$$ml\frac{d^2\theta}{dt^2} = -mg\sin\theta$$

$$\theta \to 0 \Rightarrow \sin\theta \approx \theta$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

Matematičko klatno



$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

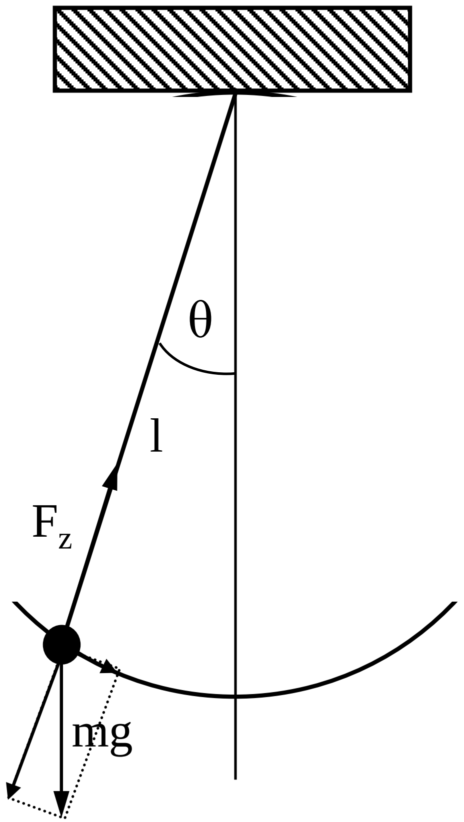
$$\theta(t) = \theta_0 \sin(\omega t + \varphi)$$

$$\frac{d\theta}{dt} = \omega\theta_0 \cos(\omega t + \varphi)$$

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta_0 \sin(\omega t + \varphi)$$

$$-\omega^2\theta_0 \sin(\omega t + \varphi) + \frac{g}{l}\theta_0 \sin(\omega t + \varphi) = 0$$

Matematičko klatno

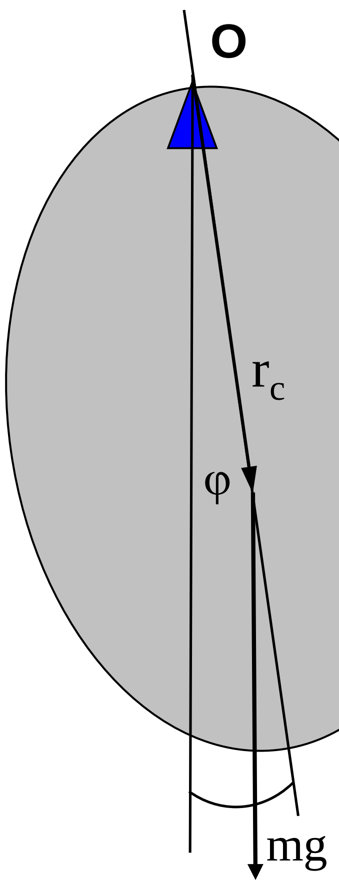


$$-\omega^2 + \frac{g}{l} = 0$$

$$\omega^2 = \frac{g}{l} \Rightarrow \omega = \sqrt{\frac{g}{l}}$$

$$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = 2\pi \sqrt{\frac{l}{g}}$$

Fizičko klatno



$$O : \left|\vec{M}\right| = \left|\vec{r}_c \times m\vec{g}\right| = mg\left|\vec{r}_c\right|\sin\varphi$$

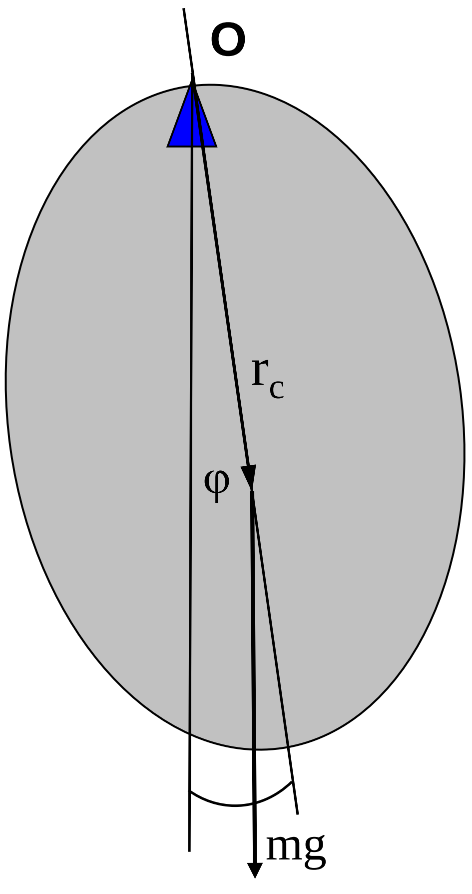
$$\left|\vec{r}_c\right| = C$$ **Rastojanje centra mase od ose rotacije**

$$I\alpha = -mgC\sin\varphi$$

$$\alpha = \frac{d^2\varphi}{dt^2} \qquad \varphi \to 0 \Rightarrow \sin\varphi \approx \varphi$$

$$\frac{d^2\varphi}{dt^2} + \frac{mgC}{I}\varphi = 0$$

Fizičko klatno



$$\varphi = Ke^{\lambda t}$$

$$\frac{d\varphi}{dt} = K\lambda e^{\lambda t} \qquad \frac{d^2\varphi}{dt^2} = K\lambda^2 e^{\lambda t}$$

$$K\lambda^2 e^{\lambda t} + \frac{mgC}{I} Ke^{\lambda t} = 0$$

$$\lambda^2 + \frac{mgC}{I} = 0 \qquad \lambda = \pm j\sqrt{\frac{mgC}{I}} = \pm j\omega$$

$$j = \sqrt{-1}$$

$$\varphi = K_1 e^{j\omega t} + K_2 e^{-j\omega t}$$

Fizičko klatno

$$\varphi = K_1 e^{j\omega t} + K_2 e^{-j\omega t} \qquad e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

$$\varphi = (K_1 + K_2)\cos(\omega t) + j(K_1 - K_2)\sin(\omega t)$$

$$(K_1 + K_2) = A \qquad j(K_1 - K_2) = B$$

$$\varphi = A\cos(\omega t) + B\sin(\omega t)$$

$$\varphi = \sqrt{A^2 + B^2}\sin\left(\omega t + arctg\frac{A}{B}\right)$$

Konstante *A* i *B* *određuju se iz početnih uslova!*

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgC}}$$

Slaganje oscilacija

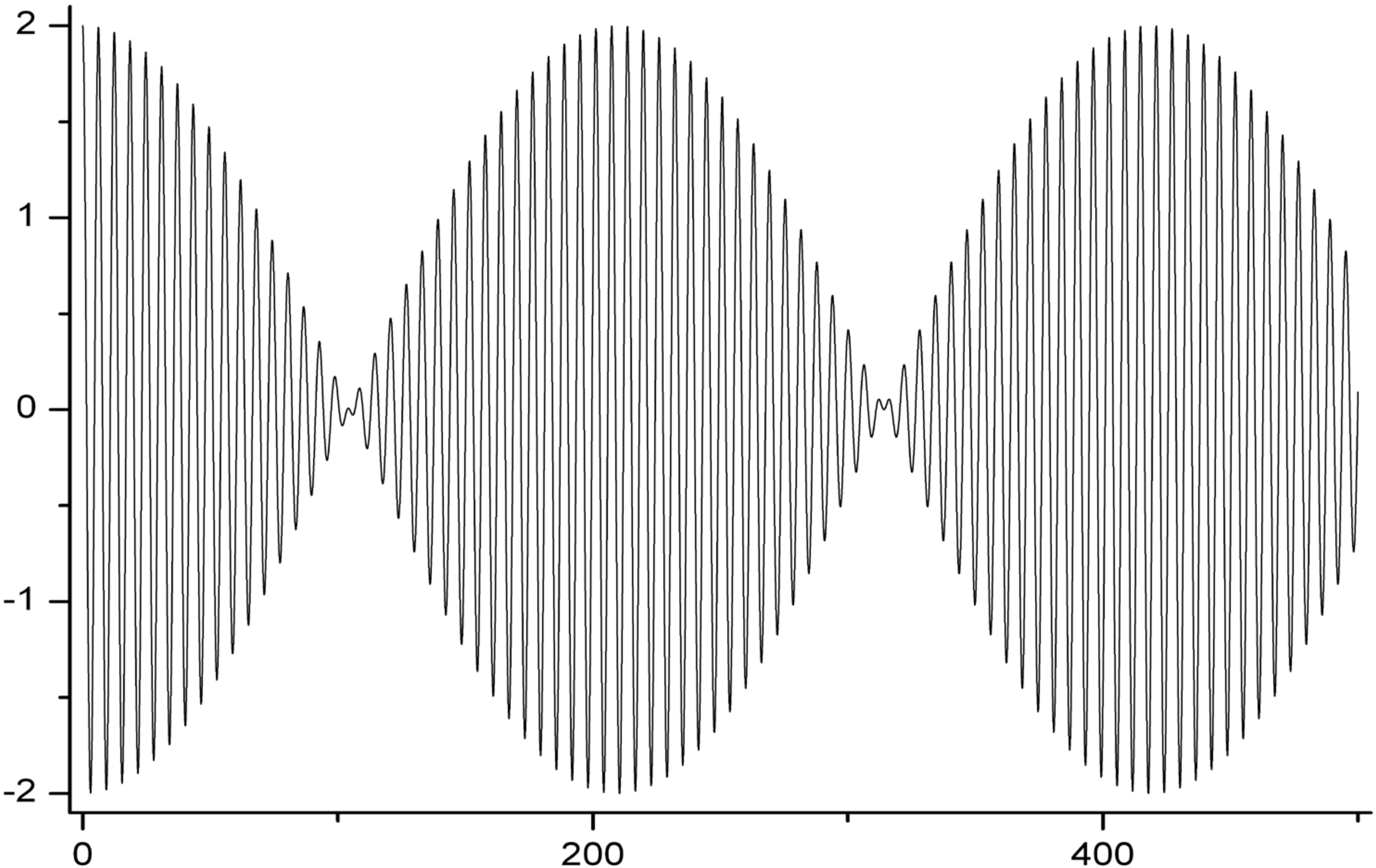
$$x_1 = A_1 \cos(\omega t + \varphi_1) \qquad x_2 = A_2 \cos(\omega t + \varphi_2)$$

$$x = x_1 + x_2 = A\cos(\omega t + \varphi)$$

$$A = \sqrt{A_1{}^2 + A_2{}^2 + 2A_1 A_2 \cos(\varphi_1 - \varphi_2)}$$

$$tg\varphi = \frac{A_1 \sin\varphi_1 + A_2 \sin\varphi_2}{A_1 \cos\varphi_1 + A_2 \cos\varphi_2}$$

Izbijanje



Lisažuove figure

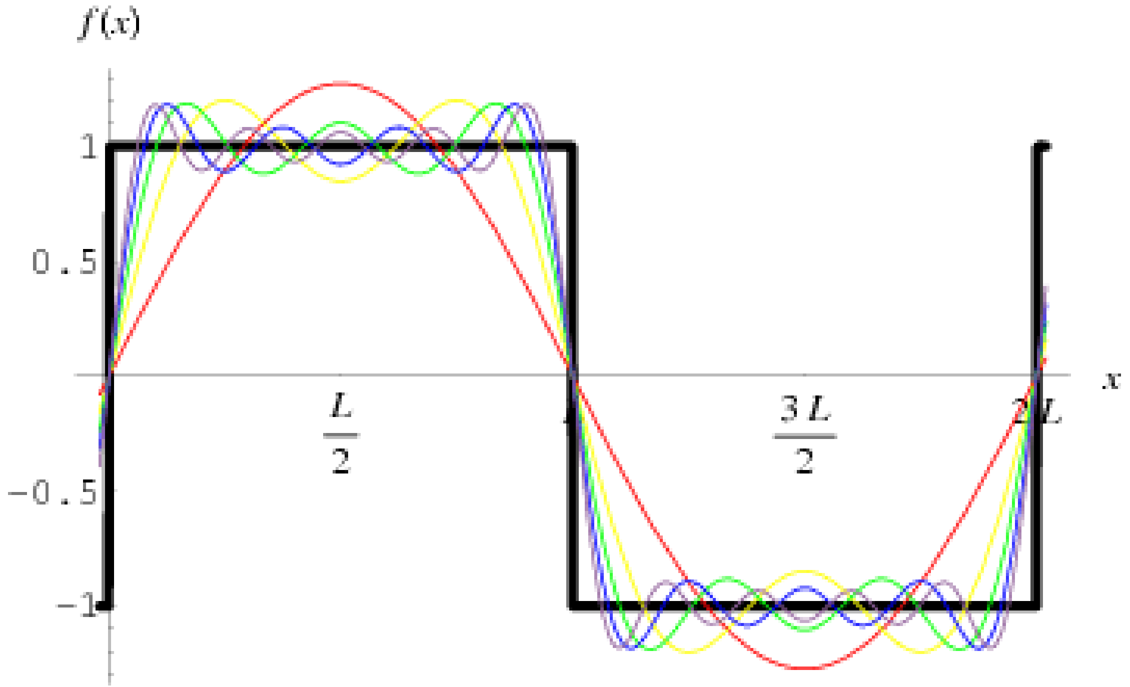
$$x = A\cos(\omega t) \qquad y = B\cos(\omega t + \varphi)$$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\varphi = \sin^2\varphi$$

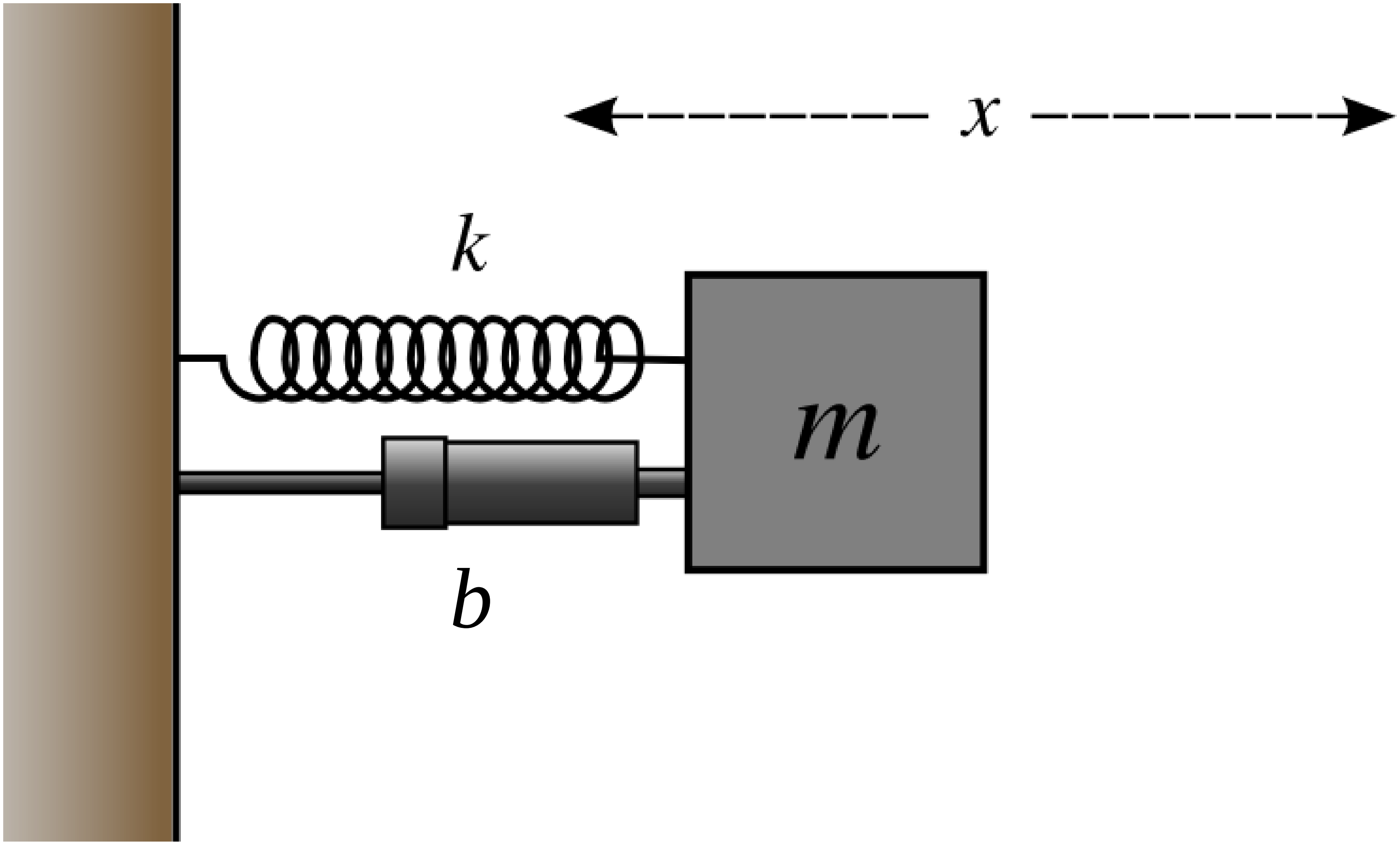
Furijeov red

$$x(t) = A_1 \sin(\omega t) + A_2 \sin(2\omega t) + A_3 \sin(3\omega t) + \ldots$$

$$+ B_1 \cos(\omega t) + B_2 \cos(2\omega t) + B_3 \cos(3\omega t) + \ldots$$



Prigušene oscilacije



Prigušene oscilacije

$$F = -kx - bv \qquad m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 \qquad \frac{b}{m} = 2\alpha \qquad \frac{k}{m} = \omega_0^2$$

$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = 0 \qquad x = Ce^{\lambda t}$$

$$\lambda^2 + 2\alpha\lambda + \omega_0^2 = 0$$

Prigušene oscilacije

$$\lambda_{1/2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$$\alpha^2 - \omega_0^2 < 0 \qquad \omega = \sqrt{\omega_0^2 - \alpha^2} \qquad \lambda_{1/2} = -\alpha \pm j\omega$$

$$x(t) = C_1 e^{(-\alpha + j\omega)t} + C_2 e^{(-\alpha - j\omega)t}$$

$$x(t) = e^{-\alpha t}\left(C_1 e^{+j\omega t} + C_2 e^{-j\omega t}\right)$$

Prigušene oscilacije

$$x(t) = e^{-\alpha t}\left(A\cos(\omega t) + B\sin(\omega t)\right)$$

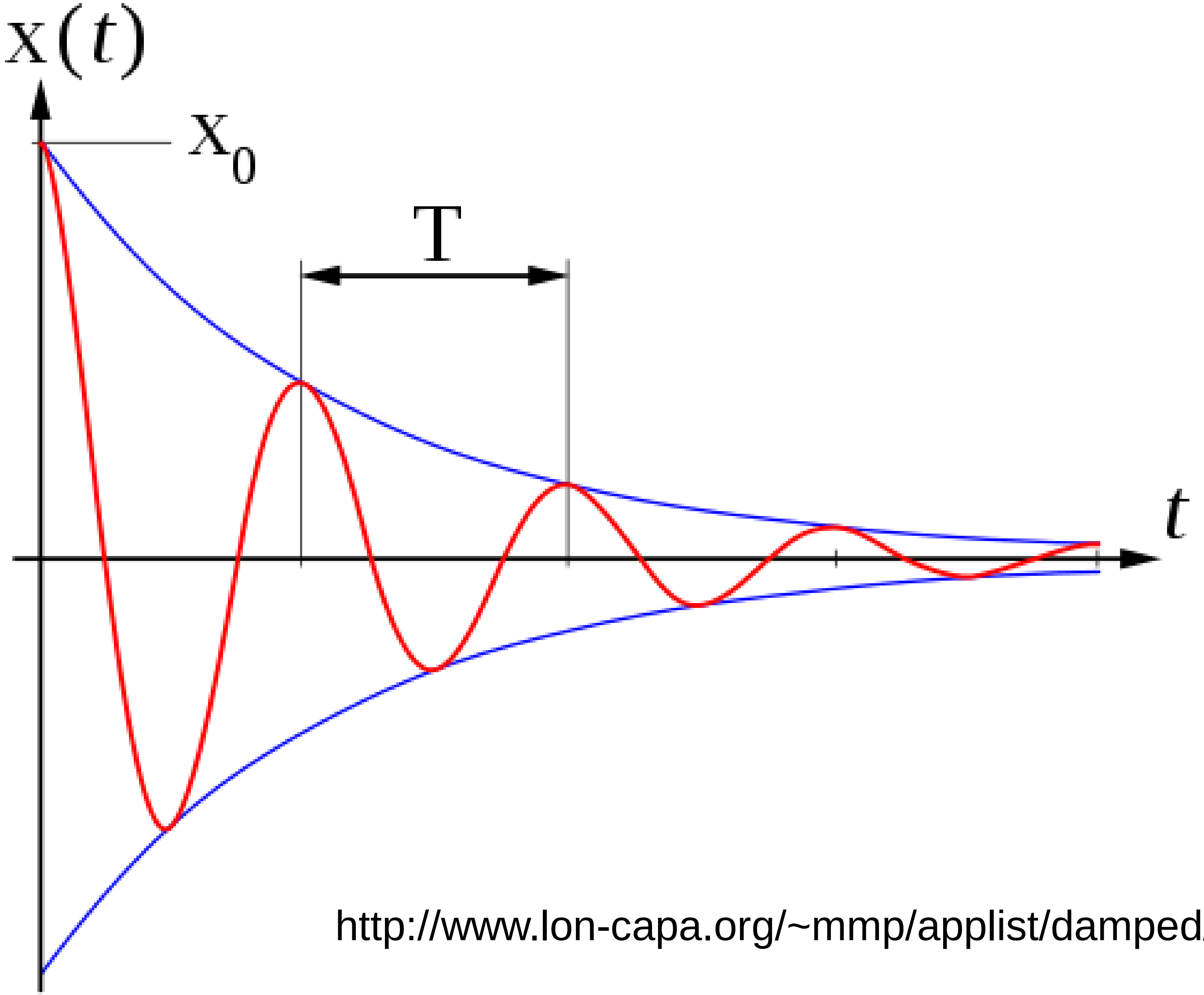
$$(C_1 + C_2) = A \qquad j(C_1 - C_2) = B$$

$$x(t) = e^{-\alpha t}\sqrt{A^2 + B^2}\,\sin\left(\omega t + arctg\,\frac{A}{B}\right)$$

$$x(t) = e^{-\alpha t}x_0 \sin(\omega t + \varphi_0)$$

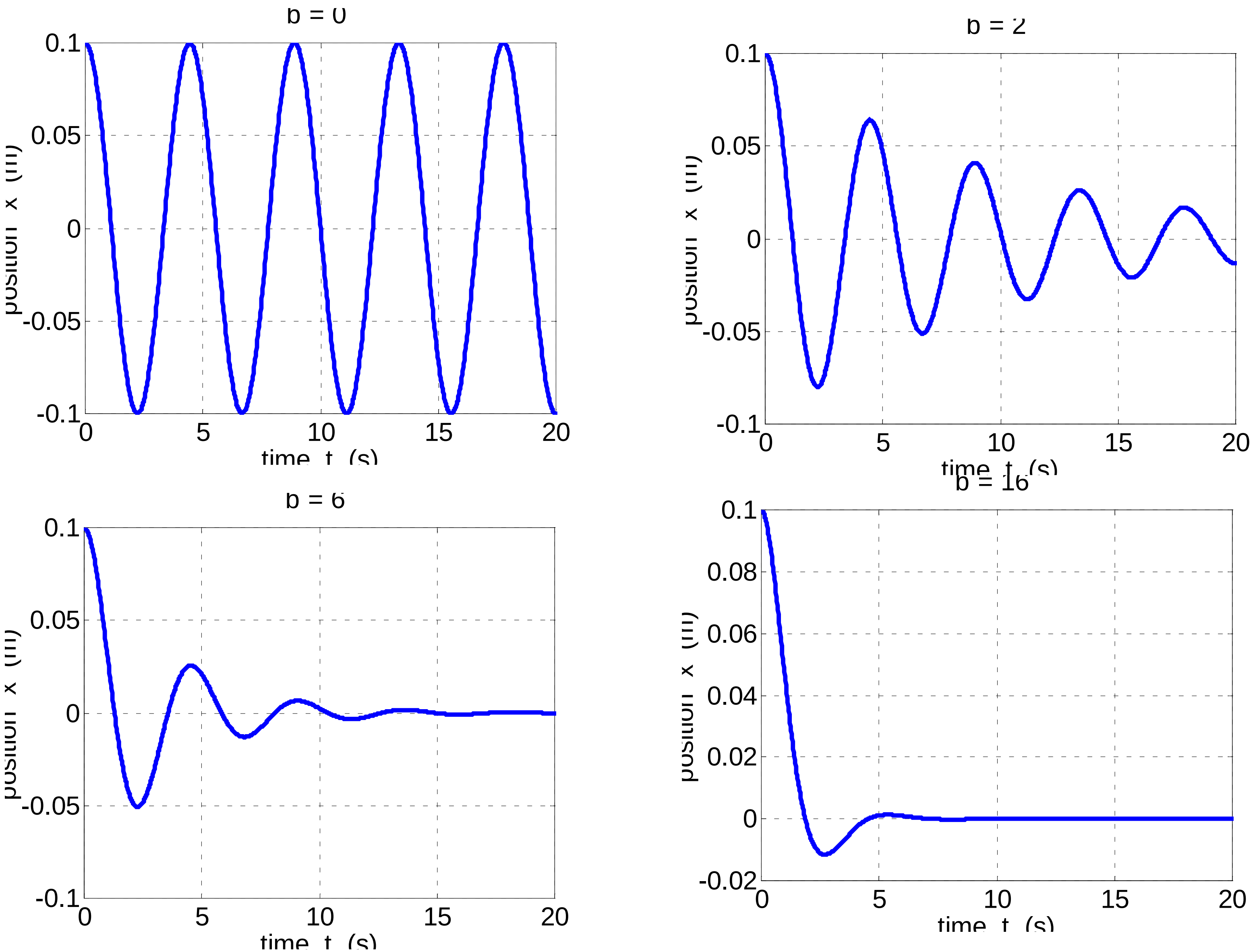
$$\omega = \sqrt{\omega_0^2 - \alpha^2} = \frac{2\pi}{T}$$

Prigušene oscilacije



http://www.lon-capa.org/~mmp/applist/damped/d.htm

Kvaziperiodično kretanje

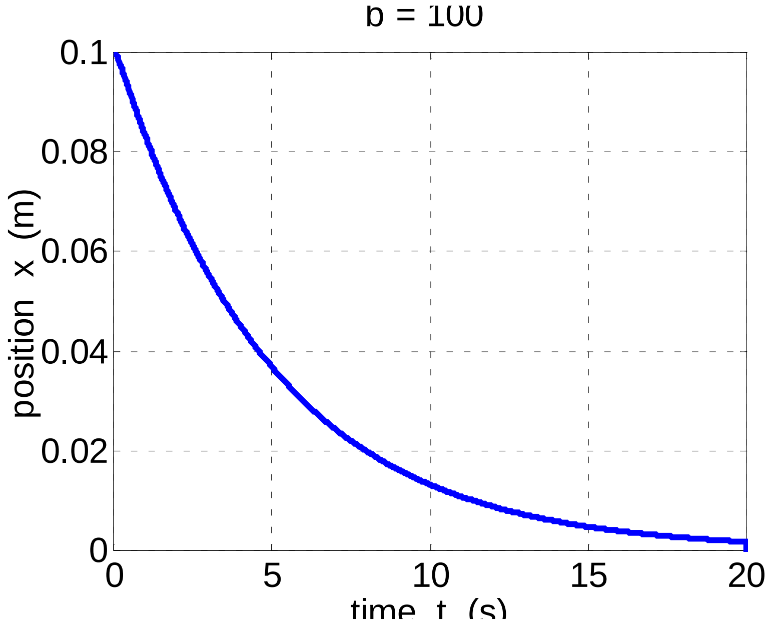


Podamortizovano

Aperiodično kretanje

$$\alpha^2 - \omega_0^2 > 0 \Rightarrow b > \sqrt{4km} \qquad \lambda_{1/2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

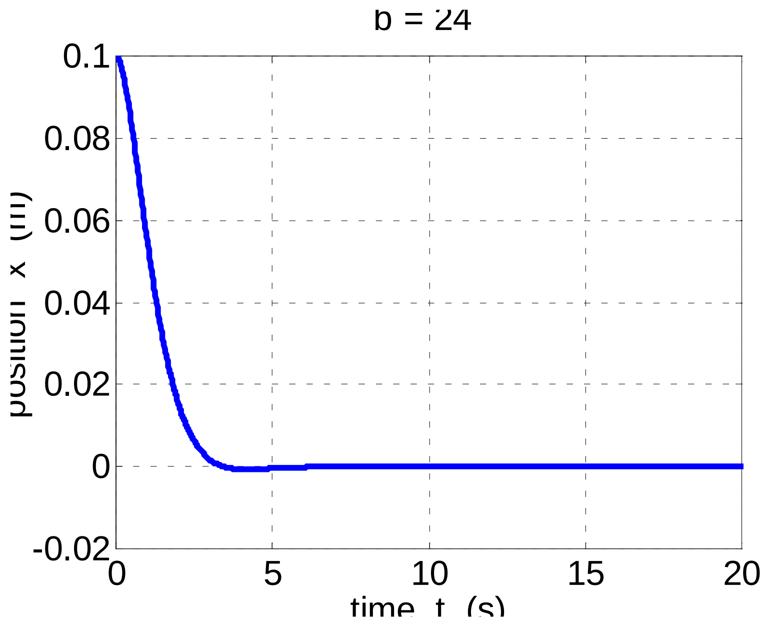


Preamortizovano

Kritično amortizovano kretanje

$$\alpha^2 - \omega_0^2 = 0 \Rightarrow b = \sqrt{4km} \qquad \lambda_1 = \lambda_2 = -\alpha$$

$$x(t) = C_1 e^{-\alpha t} + C_2 t e^{-\alpha t} = e^{-\alpha t}(C_1 + C_2 t)$$



Telo se najbrže vraća u ravnotežno stanje!

Parametri oscilatora

$$x(t) = x_0 e^{-\alpha t} \sin(\omega t + \varphi_0)$$

$$x_i = x_0 e^{-\alpha t_i} \qquad x_{i+1} = x_0 e^{-\alpha t_{i+1}}$$

$$\frac{x_i}{x_{i+1}} = e^{\alpha(t_{i+1} - t_i)} = e^{\alpha T}$$

$$\delta \equiv \ln\left(\frac{x_i}{x_{i+1}}\right) = \alpha T$$

Logaritamski dekrement

Faktor dobrote (Q-faktor)

$$Q \equiv \frac{E_i}{E_i - E_{i+1}} = \frac{E_i}{\Delta E_i}$$

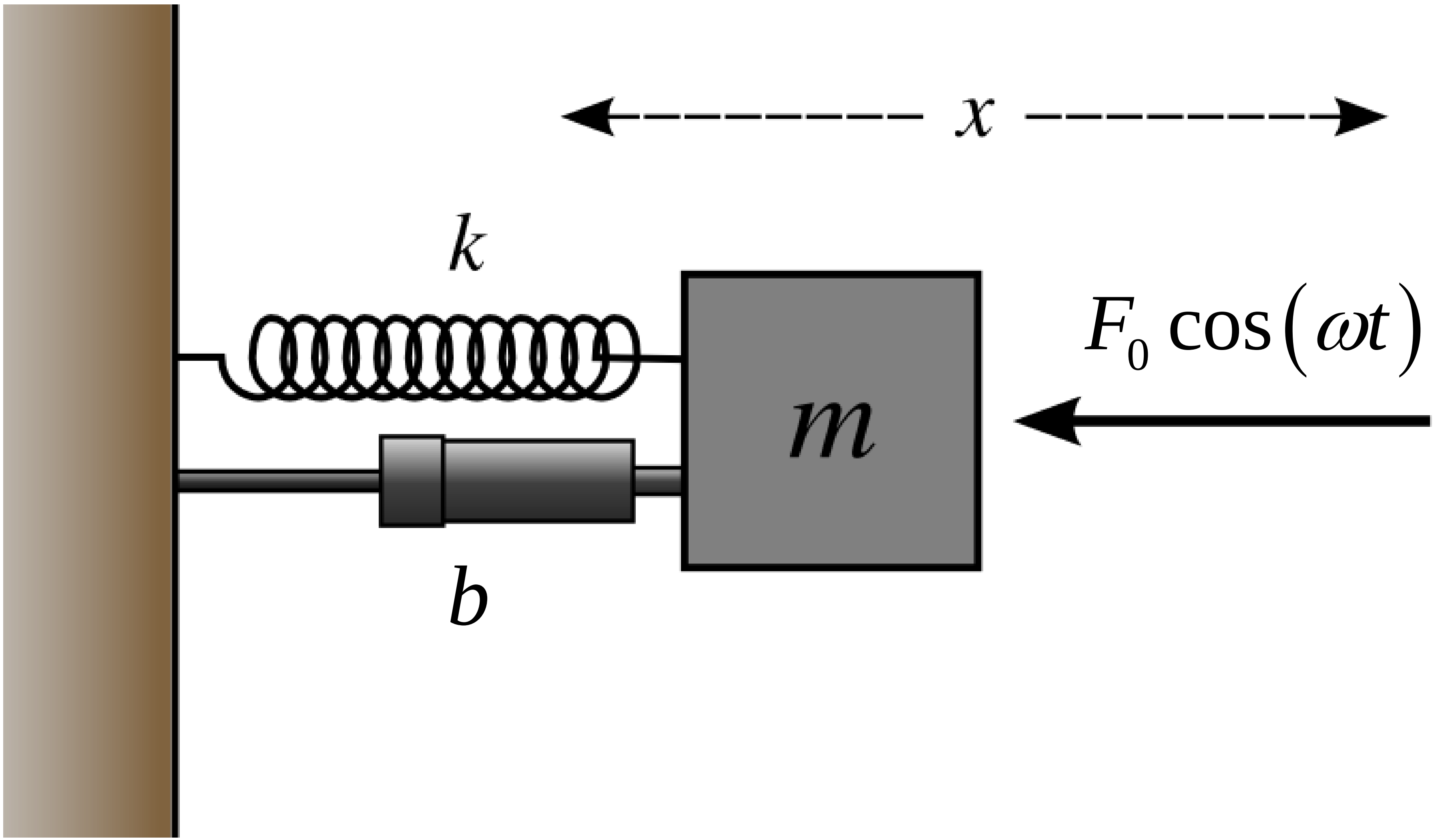
Odnos energije i gubitka energije

$$E_i = \frac{kx_i^2}{2} = \frac{kx_0^2 e^{-2\alpha t_i}}{2}$$

$$Q = \frac{e^{-2\alpha t_i}}{e^{-2\alpha t_i} - e^{-2\alpha t_{i+1}}} = \left(1 - e^{2\alpha\left(t_i - t_{i+1}\right)}\right)^{-1} = \left(1 - e^{2\alpha T}\right)^{-1}$$

$$Q = \frac{1}{1 - e^{2\delta}} \qquad \delta \to 0 \Rightarrow Q \approx \frac{1}{2\delta}$$

Prinudne oscilacije



http://www.walter-fendt.de/ph14e/resonance.htm

Prinudne oscilacije

$$F = -kx - bv + F_0 \cos(\omega t)$$

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \cos(\omega t)$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = F_0 \cos(\omega t)$$

$$\frac{b}{m} = 2\alpha \qquad \frac{k}{m} = \omega_0^2 \qquad f_0 = \frac{F_0}{m}$$

$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = f_0 \cos(\omega t)$$

Prinudne oscilacije

$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = 0 \Rightarrow x_h$$

$$f_0\cos(\omega t) \Rightarrow x_p$$

$$x(t) = x_h(t) + x_p(t)$$

$$x_h(t) = x_0 e^{-\alpha t}\sin\left(\sqrt{\omega_0^2 - \alpha^2}\,t + \varphi_0\right)$$

$$x_p(t) = A\cos(\omega t - \varphi)$$

Prinudne oscilacije

$$x_h(t) = x_0 e^{-\alpha t} \sin\left(\sqrt{\omega_0^2 - \alpha^2}\, t + \varphi_0\right)$$

$$t \to \infty \Rightarrow x_h \to 0$$

$$x_p(t) = A\cos(\omega t - \varphi)$$

$$t \to \infty \Rightarrow x_p \neq 0$$

Prinudne oscilacije

$$\frac{dx_p}{dt} = -A\omega \sin(\omega t - \varphi)$$

$$\frac{d^2 x_p}{dt^2} = -A\omega^2 \cos(\omega t - \varphi)$$

$$-A\omega^2 \cos(\omega t - \varphi) + 2\alpha\left(-A\omega \sin(\omega t - \varphi)\right) +$$

$$\omega_0^2 A \cos(\omega t - \varphi) = f_0 \cos(\omega t)$$

Prinudne oscilacije

$$-A\omega^2\cos(\omega t)\cos\varphi - A\omega^2\sin(\omega t)\sin\varphi +$$

$$+2\alpha A\omega\cos(\omega t)\sin(\varphi) - 2\alpha A\omega\cos(\varphi)\sin(\omega t) +$$

$$+\omega_0^2 A\cos(\omega t)\cos\varphi + \omega_0^2 A\sin(\omega t)\sin\varphi = f_0\cos(\omega t)$$

$$\Downarrow$$

$$-A\omega^2\cos\varphi + 2\alpha A\omega\sin(\varphi) + \omega_0^2 A\cos\varphi = f_0 \qquad (1$$

$$-A\omega^2\sin\varphi - 2\alpha A\omega\cos(\varphi) + \omega_0^2 A\sin\varphi = 0 \qquad (2$$

Prinudne oscilacije

$$(2 \Rightarrow tg\varphi = \frac{2\alpha\omega}{\omega_0^2 - \omega^2}$$

$$\varphi = arctg\left(\frac{2\alpha\omega}{\omega_0^2 - \omega^2}\right)$$ Fazno kašnjenje

$$(1 \sin(\varphi) + (2 \cos(\varphi) \Rightarrow f_0 \sin(\varphi) = 2\alpha A\omega$$

$$\sin(\varphi) = \frac{tg(\varphi)}{\sqrt{1 + tg^2(\varphi)}}$$

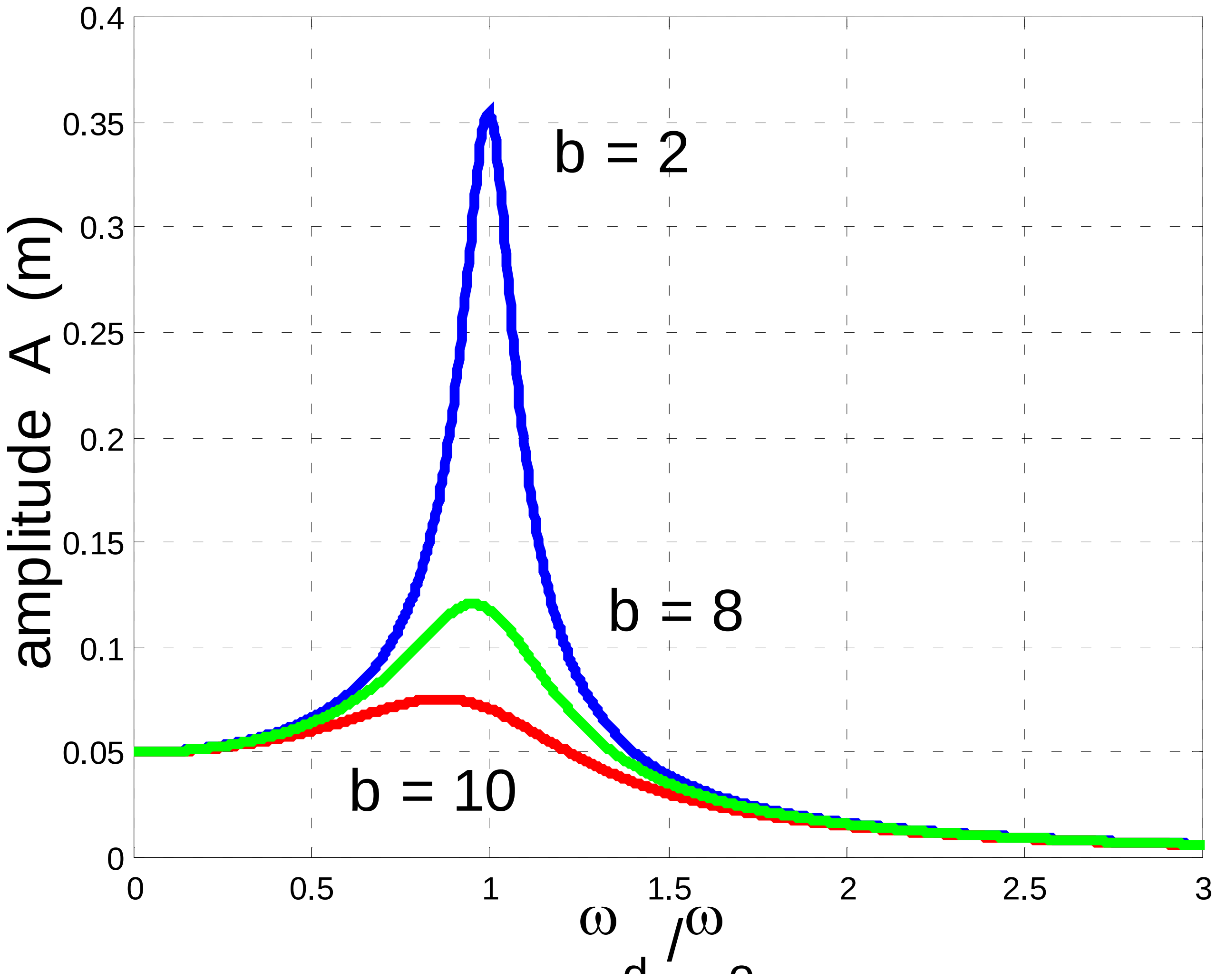
Amplituda kod prinudnih oscilacija

$$A = \frac{f_0}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\alpha^2\omega^2}}$$

$$\omega \to 0 \Rightarrow A = \frac{f_0}{\omega_0^2} = \frac{F_0}{k}$$

$$\omega \to \infty \Rightarrow A \to 0$$

Amplituda kod prinudnih oscilacija



Rezonancija

$$A \to \max$$

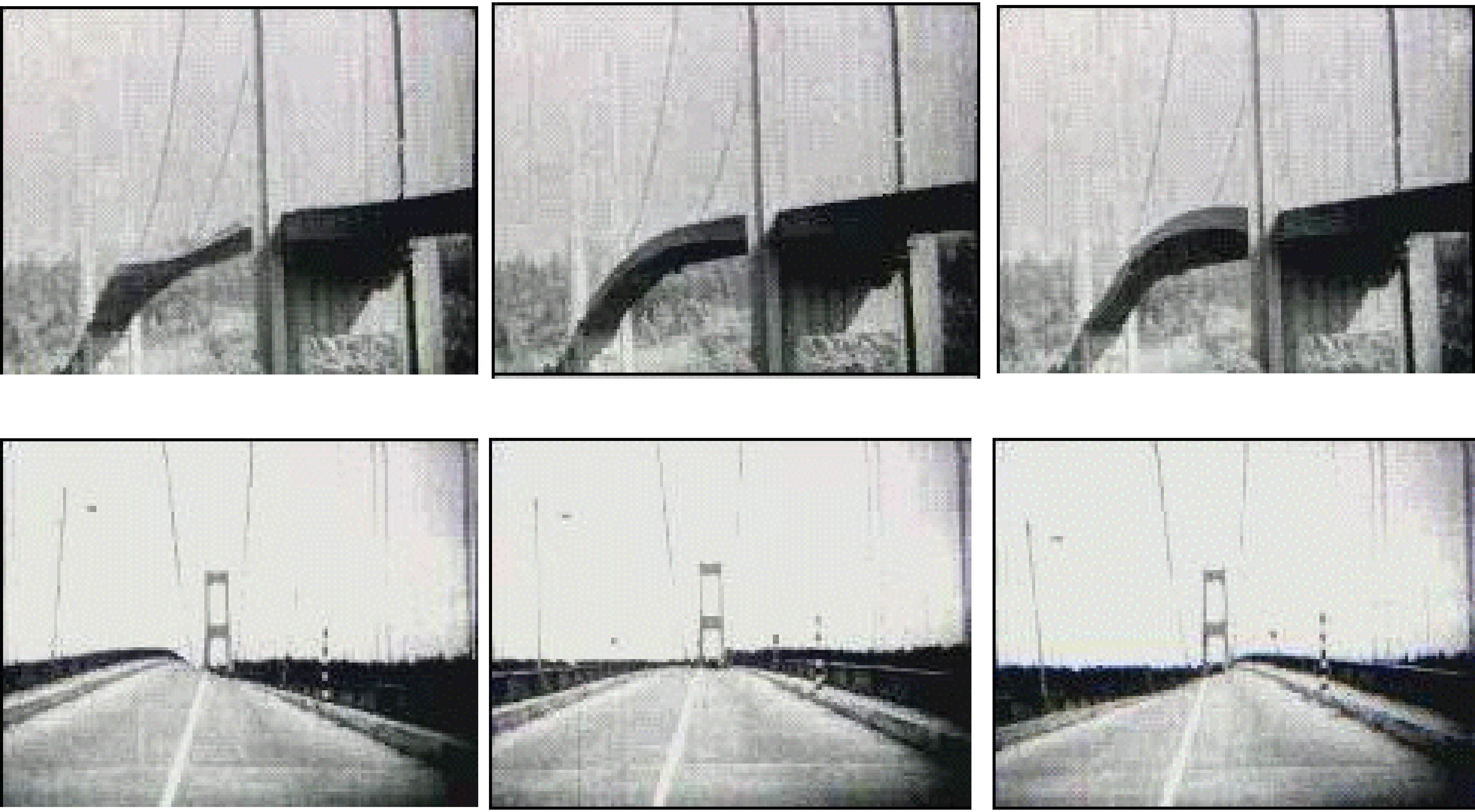
$$\frac{dA}{d\omega} = 0 \Rightarrow 2\left(\omega_0^2 - \omega^2\right)\left(-2\omega\right) + 8\alpha^2\omega = 0$$

$$\omega = \sqrt{\omega_0^2 - 2\alpha^2} = \omega_{REZ}$$

$$A\left(\omega_{REZ}\right) = \frac{f_0}{2\alpha\sqrt{\omega_0^2 - \alpha^2}}$$

$$\alpha \to 0 \Rightarrow A\left(\omega_{REZ}\right) \to \infty$$

Rezonancija (Tacoma bridge)



Značenje Q-faktora

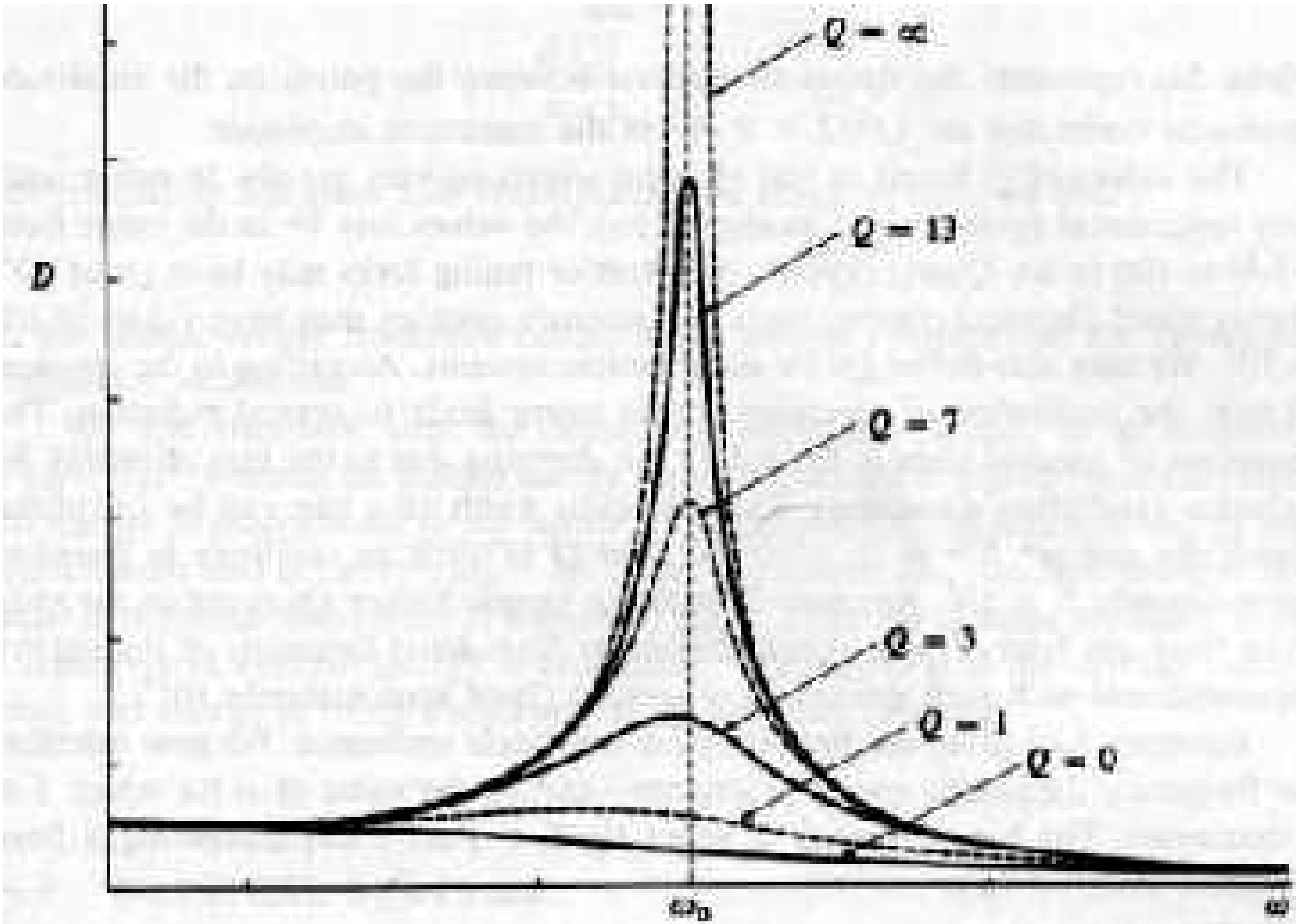
$$A\left(\omega_{REZ}\right)=\frac{f_0}{2\alpha\sqrt{\omega_0^2-\alpha^2}}$$

$$\alpha \ll \omega_0 \Rightarrow A\left(\omega_{REZ}\right)\approx\frac{f_0}{2\alpha\omega_0}$$

$$A\left(\omega=0\right)=\frac{f_0}{\omega_0^2}=\frac{F_0}{k}$$

$$\frac{A\left(\omega_{REZ}\right)}{A\left(\omega=0\right)}\approx\frac{\omega_0}{2\alpha}=\frac{2\pi}{2\alpha T}=\frac{2\pi}{2\delta}=2\pi Q$$

Amplituda kod prinudnih oscilacija



Objekat može da osciluje oko

1. bilo koje ravnotežne tačke.

2. bilo koje tačke stabilne ravnoteže.

3. bilo koje tačke, ako na njega deluje sila linearna sa rastojanjem.

4. bilo koje tačke .

Šta je neophodno da bi objekat oscilovao?

1. stabilna ravnoteža

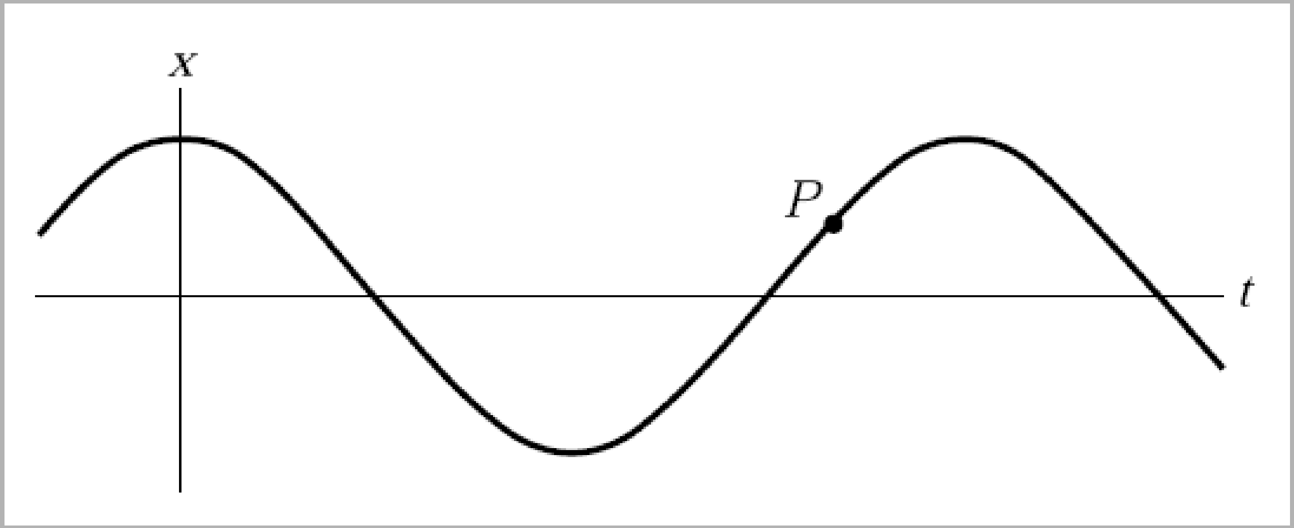
2. malo ili nimalo trenja

3. poremećaj

4. ni jedan od ponuđenih odgovora

5. prva tri ponuđena odgovora

Teg povezan sa oprugom osciluje. Na grafiku je prikazana pozicija u funkciji vremena tega. U tački P teg ima



1. pozitivnu brzinu i pozitivno ubrzanje.

2. pozitivnu brzinu i negativno ubrzanje.

3. pozitivnu brzinu i ubrzanje nula.

4. negativnu brzinu i pozitivno ubrzanje.

5. negativnu brzinu i negativno ubrzanje.

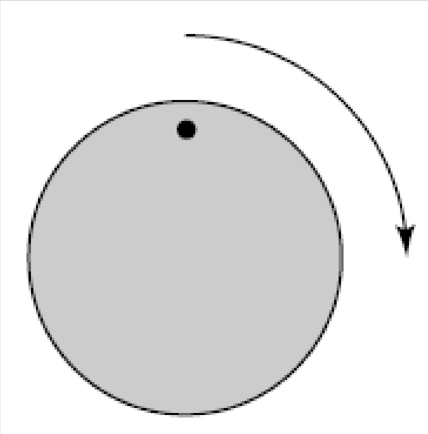
Ljuljaška se ljulja na sopstvenoj frekvenciji. Kada dete sedne na ljuljašku sopstvena frekvencija

1. se poveća.

2. ostane ista.

3. se smanji.

Krug rotira u CW smeru brzinom 29 obrta u sekundi. Krug se snima kamerom koja snima 30 slika u sekundi. Na snimljenom filmu tačka se kreće



1. veoma sporo u CW smeru.

2. veoma sporo u CCW smeru.

3. veoma brzo u CW smeru.

4. veoma brzo u CCW smeru.

5. na slučajan način

Starinski sat meri vreme pomoću klatna. Šta je od ponuđenih iskaza tačno?

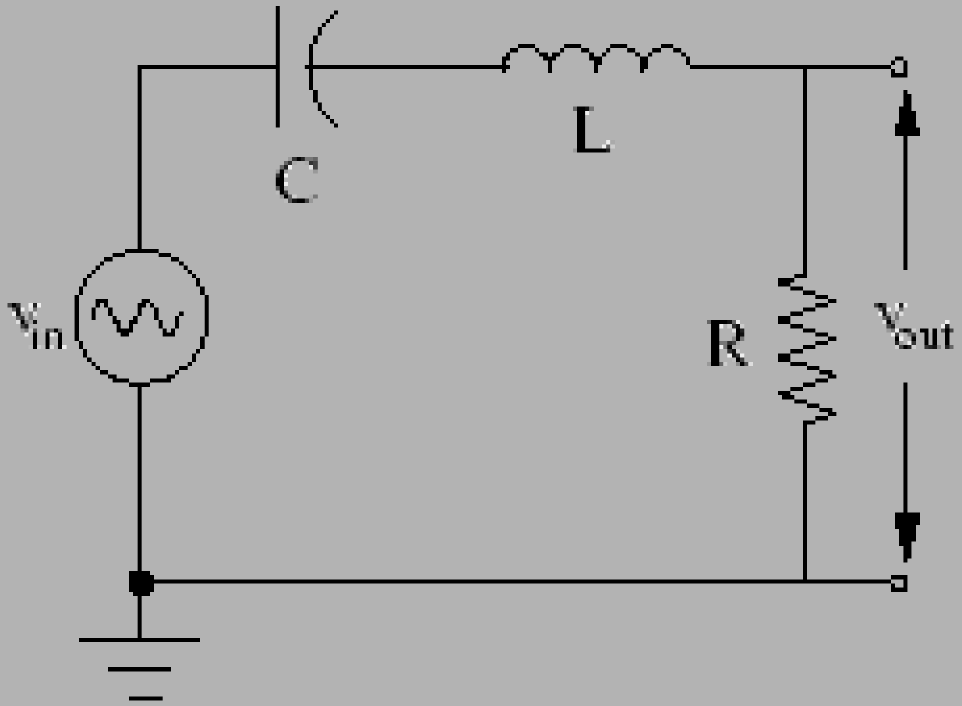
1. Sat će žuriti jer se klatnu menja period usled otporne sile.

2. Sat će žuriti ako se nalazi u brzom vozu.

3. Sat će kasniti ako se nalazi u magnetnom polju.

4. Sat će biti tačan ako je amplituda klatna dobro odabrana.

RLC kolo na slici ima veoma malu vrednost otpornosti R. Ukoliko se povećava frekvencija generatora do veoma visoke vrednosti:



1. sigurno će pregoreti kalem.
2. sigurno će probiti kondenzator.
3. sigurno će pregoreti otpornik.
4. napon na otporniku će monotono rasti.
5. nijedan iskaz nije tačan.