2018. I кол.

(1)

1. $\vec{r}(t) = At \cdot \vec{e_x} + \frac{Bt^3}{3} \vec{e_y}$ $\qquad A, B > 0 \qquad t \geq 0$

1) $x(t) = At \qquad y(t) = \frac{1}{3} B t^3$ ово је већ једначина трајекторије у параметарском облику

$t = \frac{x}{A}$

$y = \frac{B}{3} \cdot \frac{x^3}{A^3}$

$y(x) = \frac{B}{3A^3} \cdot x^3$

опрез:

$t \geq 0 \Rightarrow x \geq 0$

трајекторија је део функције $y = \frac{B}{3A^3} x^3$

2) $v_x = \dot{x} = A$

$v_y = \dot{y} = Bt^2$

$\vec{v}(t) = A \cdot \vec{e_x} + Bt^2 \vec{e_y}$

$a_x = \dot{v}_x = 0$

$a_y = \dot{v}_y = 2Bt$

$\vec{a} = 0 \cdot \vec{e_x} + 2Bt \vec{e_y}$

3) $\measuredangle \vec{v}, \vec{a} = \varphi$

$\vec{v} \cdot \vec{a} = |\vec{v}| \cdot |\vec{a}| \cdot \cos\varphi$

$\vec{v} \cdot \vec{a} = v_x a_x + v_y a_y$

$A \cdot 0 + (Bt^2) \cdot \cancel{2Bt} = \sqrt{A^2 + B^2 t^4} \cdot \cancel{2Bt} \cdot \cos\varphi$

$Bt^2 = \sqrt{A^2 + B^2 t^4} \cdot \cos\varphi$

$\cos\varphi = \frac{1}{\sqrt{2}}$

$$Bt^2 \cdot \sqrt{2} = \sqrt{A^2 + B^2t^4} \Rightarrow \cancel{2}B^2t^4 = A^2 + \cancel{B^2t^4}$$

$$B^2t^4 = A^2 \qquad t = \sqrt{\frac{A}{B}} = 1s$$

4)

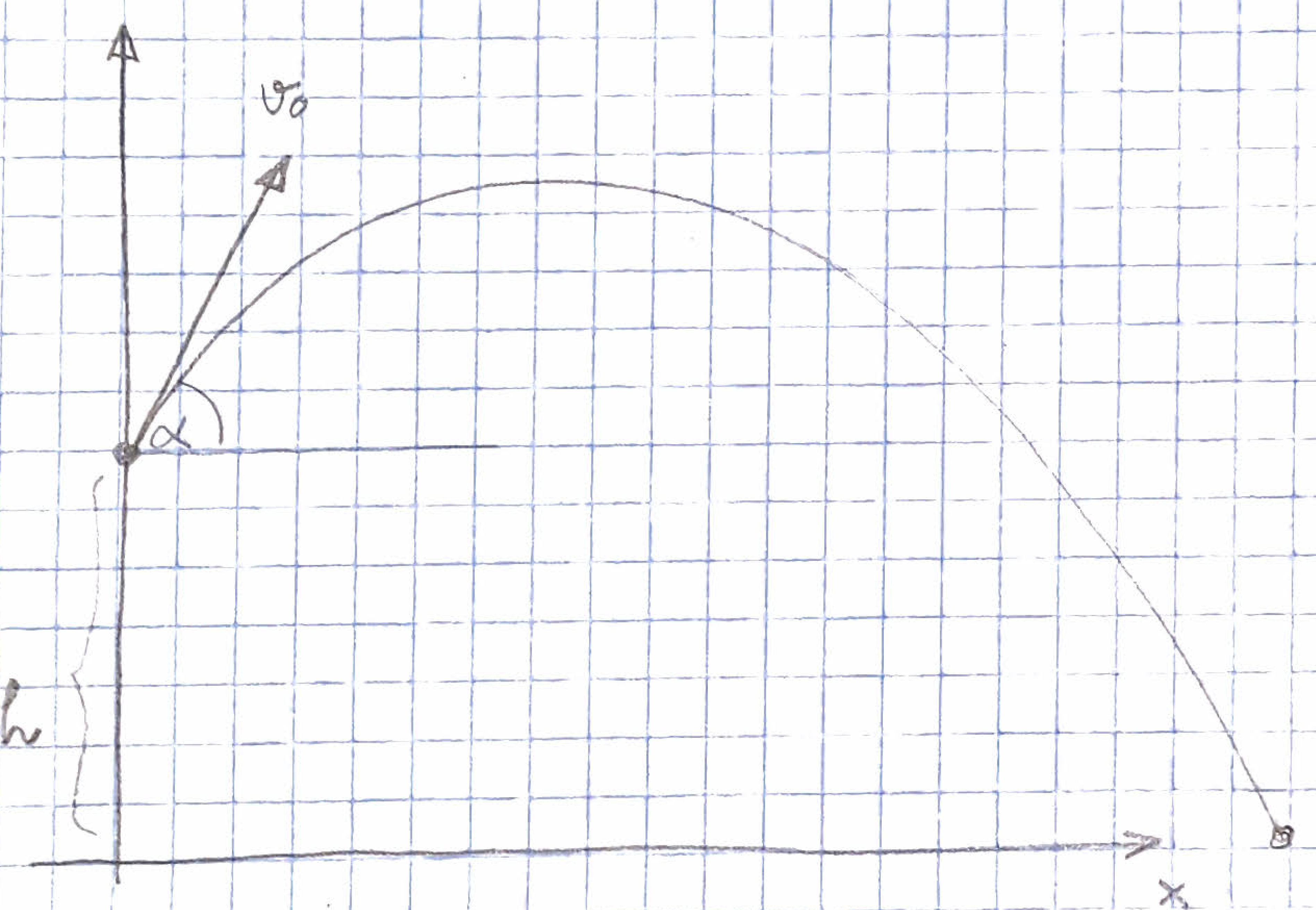
$$\langle \vec{v} \rangle_{(0,T)} = \frac{1}{T-0} \int_0^T \underbrace{\vec{v} \cdot dt}_{d\vec{r}} \Rightarrow$$

$$\underline{\underline{B=A}}$$

$$\langle \vec{v} \rangle_{(0,1)} = \int_0^1 (A\vec{e_x} + At^2\vec{e_y})\,dt =$$

$$= A\vec{e_x} \int_0^1 dt + A\vec{e_y} \int_0^1 t^2 dt = A\vec{e_x} + A\vec{e_y} \cdot \frac{1}{3}$$

2.



$\alpha = 60°$

$v_0 = \sqrt{4gh}$

$a_x = 0$

$a_y = -g$

$v_x(t) = v_0 \cos\alpha$

$v_y(t) = v_0 \sin\alpha - gt$

$x(t) = v_0 t \cos\alpha$

$y(t) = h + v_0 t \sin\alpha - \frac{1}{2}gt^2$

1) $v_x = v_y$

$$v_0 \cos\alpha = v_0 \sin\alpha - g \cdot T \qquad T = \frac{v_0}{g} \cdot (\sin\alpha - \cos\alpha)$$

$$T = \frac{\sqrt{4gh}}{g} \cdot \frac{\sqrt{3}-1}{\cancel{2}} \Rightarrow T = \sqrt{\frac{h}{g}} \cdot (\sqrt{3}-1)$$

$$x_M = \sqrt{4gh} \cdot \frac{1}{\cancel{2}} \cdot \sqrt{\frac{h}{\cancel{g}}} \cdot (\sqrt{3}-1) = h(\sqrt{3}-1)$$

$$y_M = h + \sqrt{4gh} \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{\frac{h}{g}} (\sqrt{3}-1) - \frac{1}{2} g \cdot \frac{h}{g} (4 - 2\sqrt{3})$$

$$y_M = h + h\cdot(3-\sqrt{3}) - h(2-\sqrt{3}) = 2h$$

$$a_\tau = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2+v_y^2}}$$

$$|\vec{v}| = \sqrt{v_x^2+v_y^2} = v$$

$$a_\tau = \frac{dv}{dt} = \frac{d}{dt}\left(\sqrt{v_x^2+v_y^2}\right) = \frac{\cancel{2}v_x\cdot\dot{v}_x + \cancel{2}v_y\cdot\dot{v}_y}{\cancel{2}\sqrt{v_x^2+v_y^2}}$$

za $t = T$ $\quad (v_x(T) = v_y(T))$

$$a_\tau = \frac{-g\cdot\cancel{v_y}}{\cancel{v_y}\sqrt{2}} = -\frac{g}{\sqrt{2}}, \qquad a_n^2 + a_\tau^2 = a^2 = a_x^2 + a_y^2$$

$$a_n^2 + \frac{g^2}{2} = 0 + g^2 \Rightarrow a_n = g/\sqrt{2}$$

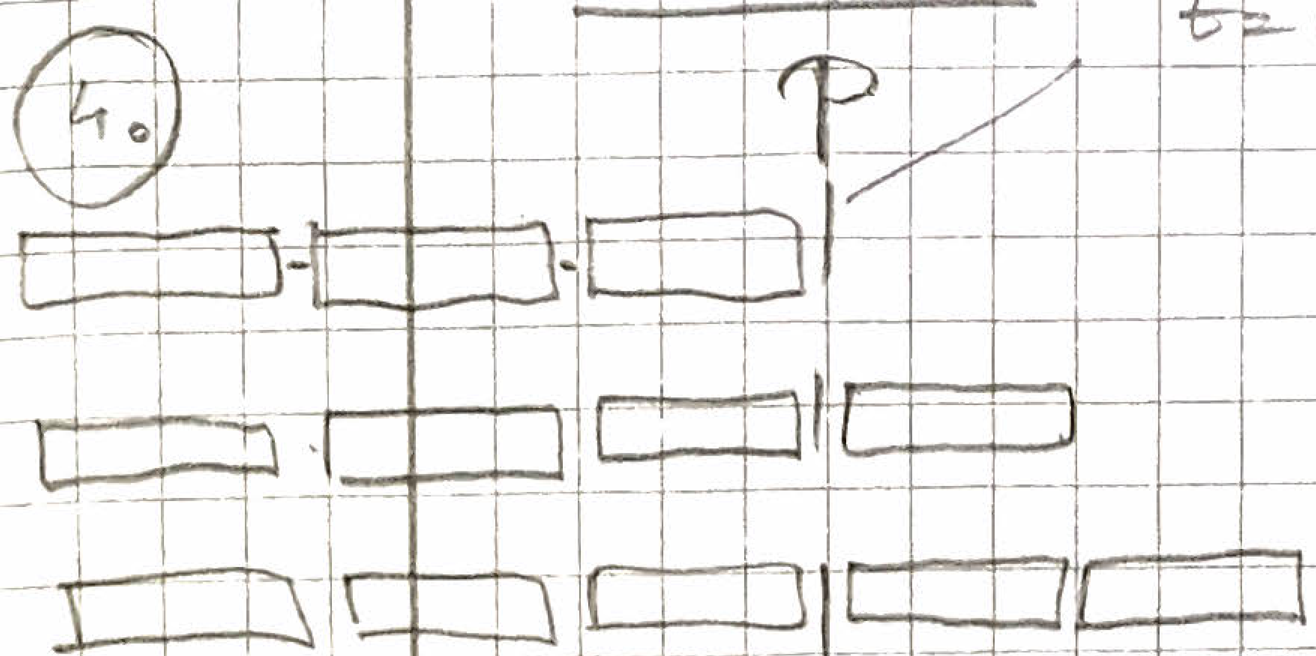
$$v_x = \sqrt{4gh}\cdot\frac{1}{2}$$

$$a_n = \frac{v^2}{R} = \frac{v_x^2+v_y^2}{R} \qquad \frac{g}{\sqrt{2}} = \frac{2v_x^2}{R}$$

$$\frac{\cancel{g}}{\sqrt{2}} = \frac{2}{R}\cdot\cancel{g}h \qquad \boxed{R = h\cdot 2\sqrt{2}} = R(T)$$

ZBIRKA:

4. $t=0$ $\quad v_0 = 0$



$t_1 = ?$

$t_1 + t_2$

$t_1 + t_2 + \dots + t_6$

$t_1 + t_2 + \dots + t_7$

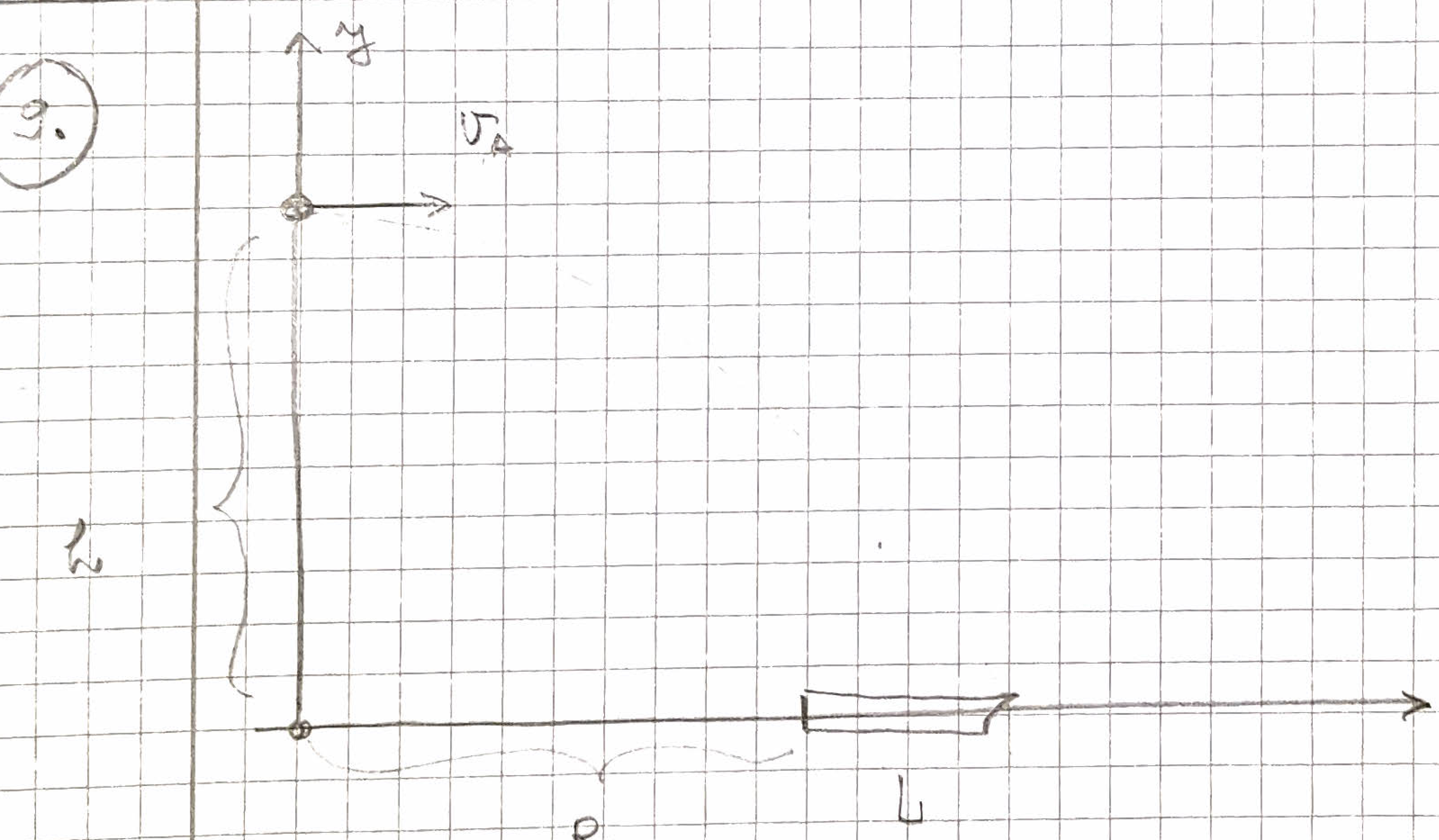
4

$$\left| \begin{array}{l} l = \frac{1}{2} a t_1^2 \\ 6l = \frac{1}{2} a (t_1 + \dots + t_6)^2 \\ 7l = \frac{1}{2} a (t_1 + \dots + t_6 + t_7)^2 \end{array} \right| \qquad t_1 = \sqrt{\frac{2l}{a}}$$

$$\left.\begin{array}{l} t_1 + \dots + t_6 = \sqrt{6} \cdot \sqrt{\frac{2l}{a}} = \sqrt{6}\, t_1 \\ t_1 + \dots + t_6 + t_7 = \sqrt{7} \cdot \sqrt{\frac{2l}{a}} = \sqrt{7}\, t_1 \end{array}\right\} -$$

$$t_7 = (\sqrt{7} - \sqrt{6})\, t_1 \qquad t_1 = \frac{t_7}{\sqrt{7} - \sqrt{6}} = t_7 (\sqrt{7} + \sqrt{6})$$

9.



$$x_p(t) = v_A \cdot t \qquad x_k(t) = R + v_B t$$

$$y_p(t) = h - \frac{1}{2} g t^2 \qquad y_k(t) = 0$$

нека t_0 кад падне ;

$$x_p(t_0) = v_A \cdot t_0 \qquad t_0 = \sqrt{\frac{2h}{g}}$$

$$y_p(t_0) = h - \frac{1}{2} g t_0^2 = y_k(t_0) = 0$$

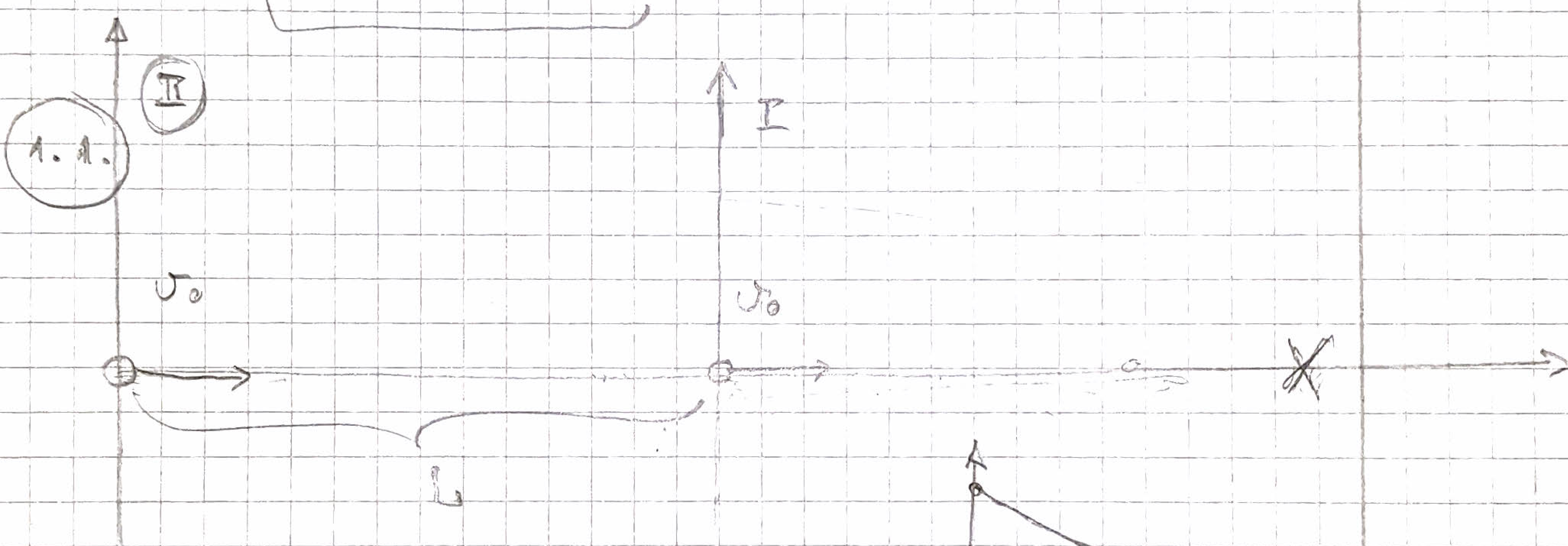
$$x_p(t_0) = v_A \cdot \sqrt{\frac{2h}{g}}$$

услов задатка $= \quad R + v_B t_0 \leq x_p(t_0) \leq R + v_B t_0 + L$

$$R + v_B\sqrt{\frac{2h}{g}} \leq v_A \cdot \sqrt{\frac{2h}{g}} \leq R + L + v_B \cdot \sqrt{\frac{2h}{g}}$$

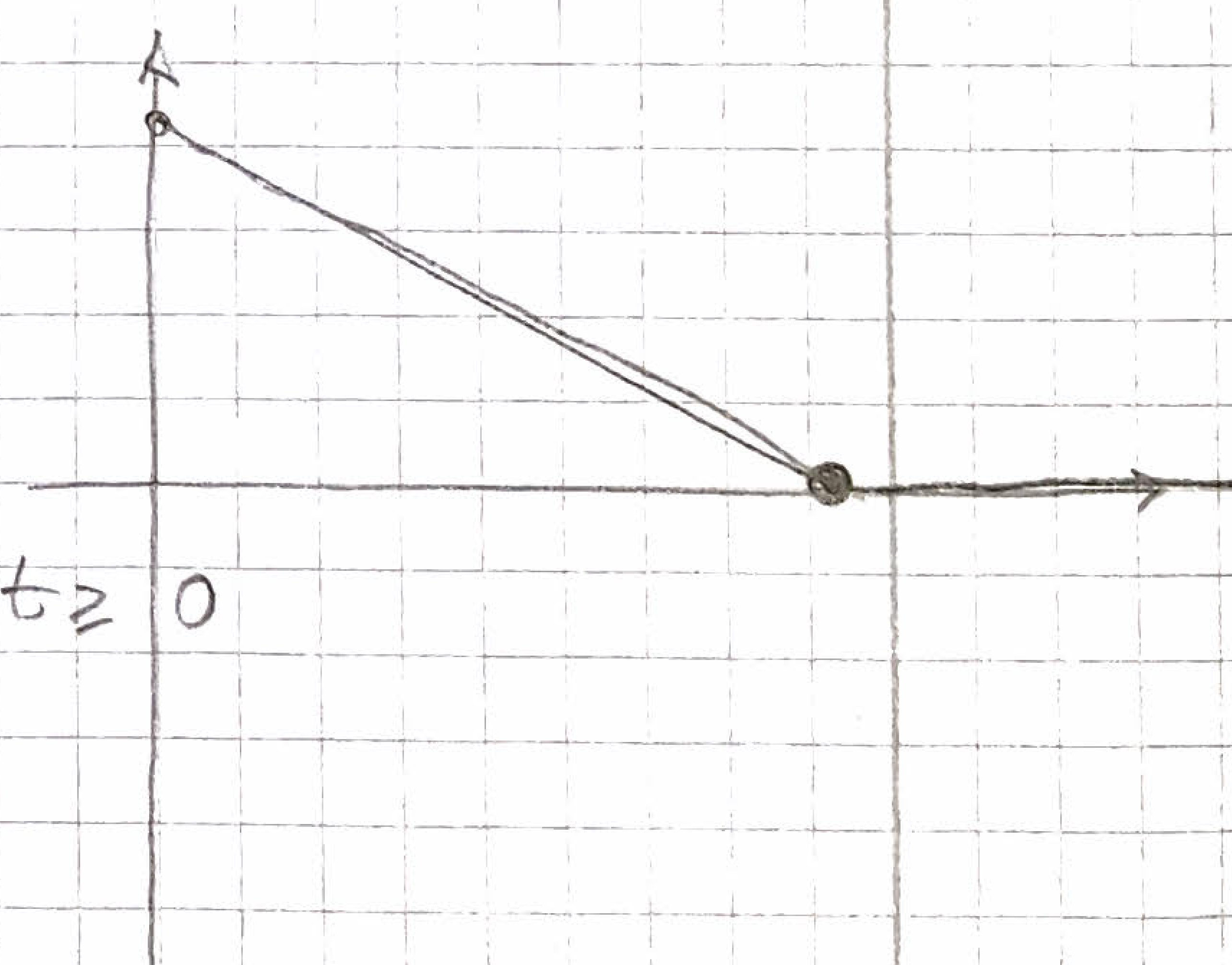
$$v_B + R\sqrt{\frac{g}{2h}} \leq v_A \leq v_B + (R+L) \cdot \sqrt{\frac{g}{2h}}$$

12. II 2019.



I

$$\left.\begin{aligned} v_1 &= v_0 - a \cdot t \\ x_1 &= v_0 t - \frac{1}{2}at^2 + L \end{aligned}\right\} \quad t \geq 0$$



II

$$v_2 = \begin{cases} v_0 & 0 \leq t \leq T \\ v_0 - a(t-T) & t \geq T \end{cases}$$

$$x_2 = \begin{cases} v_0 t & 0 \leq t \leq T \\ v_0 t - \frac{1}{2}at^2 + aT \cdot t + C_2 & t \geq T \end{cases}$$

$$\cancel{v_0 T} = \cancel{v_0 T} - \frac{1}{2}aT^2 + aT^2 + C_2 \qquad C_2 = -\frac{1}{2}a_1 T^2$$

I) $v_1 = v_0 - at$ стане после $\frac{v_0}{a}$

$t \geq 0$

$x_1 = v_0 t - \frac{1}{2}at^2 + L$

II) $v_2 = \begin{cases} v_0 & 0 \leq t \leq T \\ v_0 - a(t-T) & t \geq T \end{cases}$

$x_2 = \begin{cases} v_0 t & 0 \leq t \leq T \\ v_0 t - \frac{1}{2}a(t-T)^2 & t \geq T \end{cases}$

$x_1 > x_2$ ще се заудрят

$t = T + \frac{v_0}{a}$

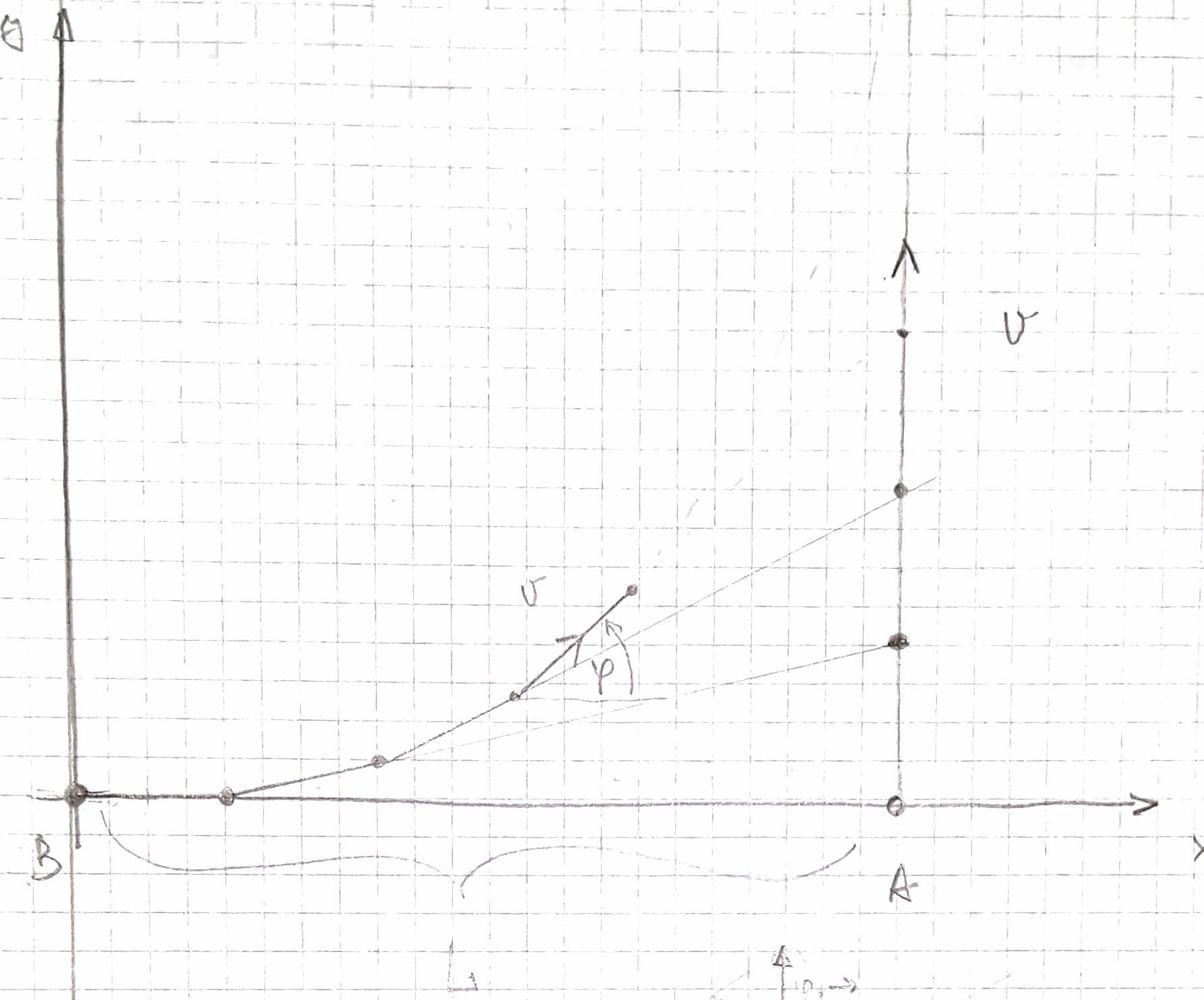
$x_1^{max} = v_0 \cdot \frac{v_0}{a} - \frac{1}{2}a \cdot \frac{v_0^2}{a^2} + L$

---

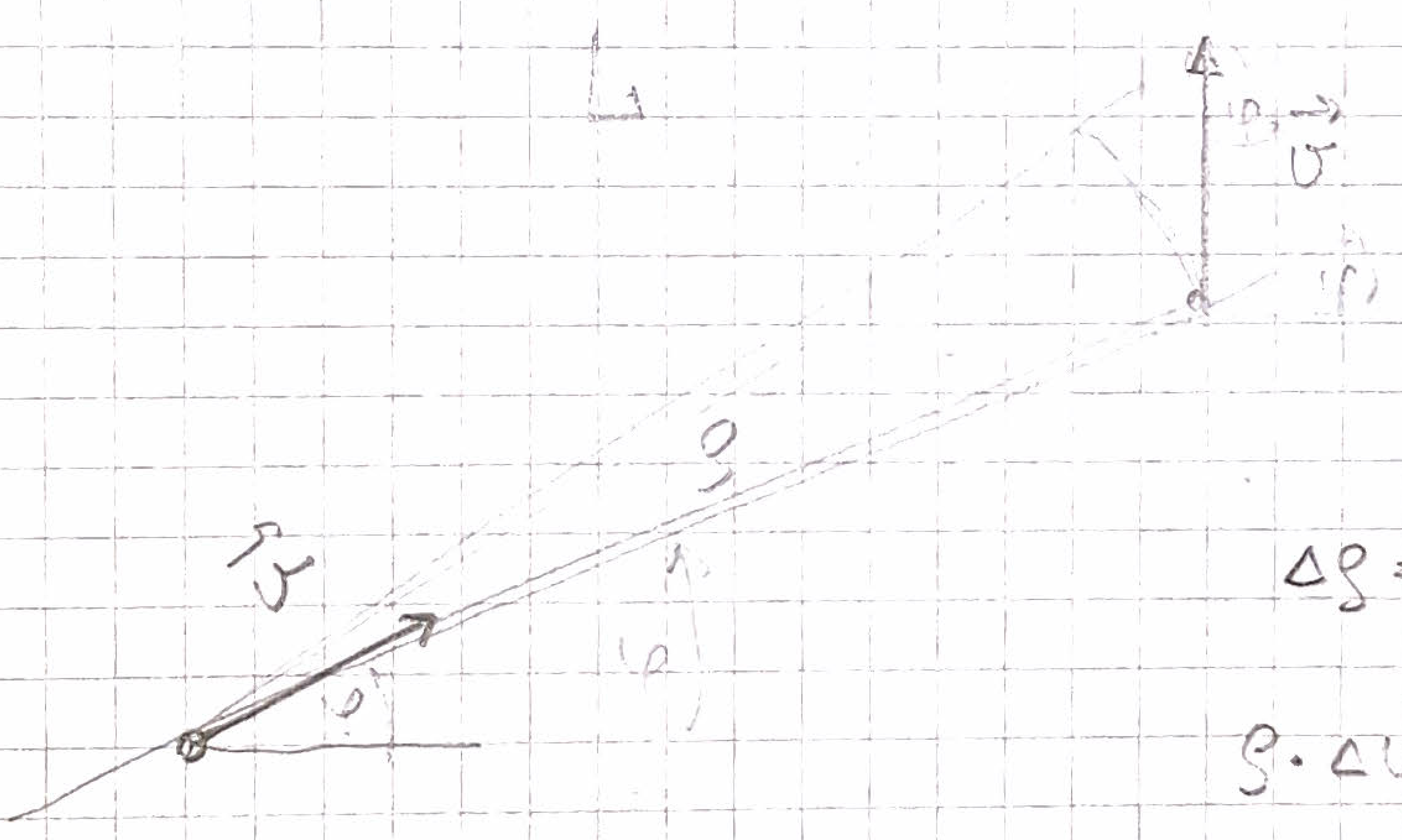
x_2 ; $t = \frac{v_0}{a} + T$

$v_0 \cdot \left(\frac{v_0}{a} + T\right) - \frac{1}{2}a \cdot \left(\frac{v_0}{a}\right)^2 = x_2 < x_1^{max}$

$v_0 T < L$



Δt



$\Delta\rho < 0$

$$\Delta\rho = v\sin\varphi \cdot \Delta t - v\Delta t$$

$$\rho \cdot \Delta\varphi = (v\cos\varphi)\,\Delta t$$

$t=0$

$\downarrow$

$\varphi(0) = 0$

$\varphi(+\infty) = \frac{\pi}{2}$

$\rho(0) = L$

$\rho(+\infty) = D = ?$

$$\frac{\Delta\rho}{\rho\Delta\varphi} = \frac{\cancel{v}\sin\varphi - \cancel{v}^{1}}{\cancel{v}\cos\varphi}$$

$$\frac{d\rho}{\rho} = \frac{\sin\varphi - 1}{\cos\varphi} \cdot d\varphi$$

$$\int_L^D \frac{d\rho}{\rho} = \int_0^{\pi/2} \frac{\sin\varphi - 1}{\cos\varphi}\, d\varphi$$

$\varphi = \frac{\pi}{2} - \theta$

$d\varphi = -d\theta$

$\theta = \frac{\pi}{2} - \varphi$

$$\ln\frac{D}{L} = -\int_{\pi/2}^{0} \frac{\cos\theta - 1}{\sin\theta} \cdot d\theta = -\int_0^{\pi/2} \frac{1-\cos\theta}{\sin\theta}\, d\theta = -\int_0^{\pi/2} \frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}\, d\theta$$

(8)

$$\ln \frac{D}{L} = -\int_0^{\pi/2} \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\, d\theta = 2 \ln \left|\cos\frac{\theta}{2}\right| \Big|_0^{\pi/2} =$$

$$= 2 \ln \frac{1}{\sqrt{2}} = \ln \frac{1}{2} \qquad \frac{D}{L} = \frac{1}{2} \qquad \underline{D = \frac{1}{2} L}$$